Colonel Blotto

Adam Shull

May 5, 2009

1 Introduction to Colonel Blotto

Colonel Blotto is a game for two players. There are C castles that the players try to capture. Each player has S soldiers that they can distribute among the castles however they want. Whoever has the most soldiers at the castle wins the castle; if they have the same number than neither player wins the castle. Castle k is worth P_k points, where P is a list of nonnegative integers. This game is commonly played with $C = 10$, $S = 100$, and $P = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$. A game played with these values will be referred to as a *standard Blotto game*.

2 Analysis of Basic Strategies for a Standard Blotto Game

There is no ideal way to allocate the soldiers in a standard Blotto game. Strategy A may defeat Strategy B, but a Strategy C can usually be created that defeats Strategy A but loses to Strategy B. For example, let Player A use $[0, 0, 0, 0, 0, 20, 20, 20, 20, 20]$ and Player B use $[10, 10, 10, 10, 10, 10, 10, 10, 10, 10]$. Player A gets 40 points while Player B only gets 15 points, so Player A wins. However, if Player C uses $[0, 0, 0, 0, 0, 0, 0, 25, 25, 50]$, then Player C defeats Player A 27-13 but Player B defeats Player C 28-27. Thus it is clear that strategies in Colonel Blotto are non-transitive, and no strategy can be created that always wins.

If two people play a standard Blotto game repeatedly and one of them always uses the same strategy, it will not take long for the other player to develop a different strategy that defeats the first one. Alternating between two or more strategies is not a much better idea, since the opponent could create a strategy that defeats most or all of the those strategies. In fact, any strategy that can be determined or approximated by previous strategies should not be used, since this allows the opponent to use a strategy specifically designed to counter the likely strategy.

3 Weighted Random Strategies

One way for players to keep their opponents from predicting their strategies is to incorporate randomness. The most basic random strategy involves distributing all the soldiers randomly so a soldier has an equal chance of going to each castle. However, this may lead to too many soldiers being used for the low-value castles, allowing the opponent to claim the high-value castles and get more points. The basic random strategy will lose to a strategy such as $[0, 0, 0, 0, 0, 20, 20, 20, 20, 20]$ practically every time.

An improved random strategy weighs the probability for each castle based on its value. This would mean that a soldier has twice the probability of going to Castle 2 as Castle 1, and five times the probability of going to Castle 10 as Castle 5. This strategy beats the basic random strategy 95 percent of the time. However, this is still relatively weak, as it only wins against $[0, 0, 0, 0, 0, 20, 20, 20, 20, 20]$ about 7 percent of the time.

There is no reason why the weights for each castle need to be linearly related to the castle's value. In fact, a much stronger strategy weighs each castle based on the square of its value. This wins against the linearly weighted strategy 65 percent of the time. It is also slightly better than a strategy based on the cubes of the values in a head-to-head match. When the quadratic and cubic strategies face each other, the quadratic strategy wins 50 percent of the time while the cubic strategy wins 45 percent of the time. The quadratically weighted strategy defeats $[0, 0, 0, 0, 0, 20, 20, 20, 20, 20]$ 68 percent of the time, while the cubically weighted strategy defeats $[0, 0, 0, 0, 0, 20, 20, 20, 20, 20]$ 88 percent of the time. The cubic strategy does better here because it allocates more soldiers to the high-value castles, allowing it to win those as well as the low-value castles. Against $[10, 10, 10, 10, 10, 10, 10, 10, 10, 10]$, the quadratic strategy wins 93 percent of the time while the cubic strategy wins 76 percent of the time. Thus the quadratically weighted strategy is the best overall, but the cubically weighted strategy comes close and is superior for certain situations.

4 Minimum Winning Sets

In order to win a match, a player does not need to contend for all the castles. The player just needs to get a set of castles whose total value is more than half of the value of all the castles. A player does not need to use soldiers on more castles than necessary, so the player should choose a set of castles where each castle is necessary in order to get a majority of points. Such a set is called a *minimum winning set.* In a standard Blotto game, the total value of the castles is 55 points so 28 points are needed to win. There are 77 minimum winning sets in a standard game.

A new type of weighted random strategy uses this idea. First a minimum winning set is selected at random. Then the weight of each castle is calculated, but castles not in the winning set are given a weight of 0. Finally soldiers are randomly allocated based on these weights. For example, a minimum winning set is Castles 4, 6, 8, and 10. A possible weight based on this set and linearly related to each castle's value is $[0, 0, 0, 14, 0, 20, 0, 30, 0, 36]$.

A minimum strategy that does not depend on the weight of each castle defeats an ordinary completely random strategy 84 percent of the time. If these strategies are weighed linearly based on the values of the castles, then the minimum strategy wins 74 percent of the time. If they are weighed quadratically then the minimum strategy wins 57 percent of the time. The quadratically weighted minimum strategy is also generally the best here, since it defeats the linear strategy 58 percent of the time and the cubic strategy 49 percent of the time while losing 47 percent of matches to the cubic strategy. The minimum quadratic strategy beats $[0, 0, 0, 0, 0, 20, 20, 20, 20, 20]$ 71 percent of the time. However, this strategy is not without its weaknesses. In fact, it loses to a completely random strategy 86 percent of the time, since all the random strategy has to do to win is usually to capture one of the castles the minimum quadratic strategy needs.

5 Conclusion

There are plenty of other ways to create strategies for standard Blotto games. One technique that works cell against certain strategies is vulnerable to other strategies, so a person might never find a completely satisfactory strategy. The lack of transitivity is what makes Colonel Blotto such an interesting game.