Tutoring Dialogue Goals for Conceptual Understanding of Differential Equations: Preliminary Work

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Abstract

The Differential Equations Tutoring Project is studying the conversational dialogue during problem-solving tutoring sessions, where the aim is for the student to grasp the solution technique in conceptual ways. In this study, we analyze the common conversational and tutorial goal structure in nine keyboard-to-keyboard tutoring sessions. We compared these goals, along with the tutoring dialogue, to a proposed framework of skills that are used for assessing whether students have a grasp of the concepts underlying the solution technique.

Introduction

In this study we dissect transcripts of students being tutored in the separation of variables technique for solving differential equations. The aim of this project is to understand how such tutoring works, as well as partially computerize the tutoring process [Glass et al., 2007]. We analyzed the tutor's conversation as structured collections of conversational goals [Grosz and Sidner, 1986], a formalism designed for dialogue analysis and planning.

In our model of how tutoring works the tutor brings to the dialogue a set of overall goals that must be achieved. For example, our previous study of algebra tutoring [Kim and Glass, 2004] found goals such as "introduce the problem," "identify the operation to be performed," "have the student perform the operation," "have the student verify the answer," and "provide acknowledgement to the student." Each of these goals can be satisfied, during the conversation, in possibly several ways. For example, in order to identify the operation the tutor might first employ a sub-goal that simply asks the student what operation is to be performed. If the student fails to give the desired answer, the tutor has other sub-goals that provide hints and assistance to the student until eventually the main goal of identifying the operation succeeds. It is only by studying a number of dialogues that we can discover a full kit of possible tutoring moves that the tutor employs.

Structured collections of dialogue and tutorial goals drive a number of modern tutorial dialogue planners, among them the APE planner [Freedman, 2000] which lies at the core of the TuTalk dialogue authoring system [Jordan et al., 2006] and AutoTutor [Graesser et al., 2005], a successful pioneering retargetable dialogue-based tutor. Kim et al. [2006] describe an elaborate set of goals for the CIRCSIM-Tutor [Evens and Michael. 2006] system for teaching medical students about the baroreceptor reflex. The Wooz software [Kim & Glass, 2004] displays a map of algebra goals as an aid to a human tutor, while Wooz-2 [Glass et al., 2007] will attempt to match up such a goal structure with a differential equations tutorial dialogue as it progresses.

Framework for Relational Understanding

Teaching for this project focuses on relational knowledge, concepts beyond the simple procedural skills of solving DEs, as described by the Inquiry-Oriented Differential Equations (IO-DE) project [Rasmussen, 2001; Rasmussen and Kwon, 2007]. Allen and Rasmussen have defined a framework consisting of six competencies that students should exhibit if they have developed a relational understanding of differential equation solution procedures [Keene, 2009]. What distinguishes this framework is that it is assessable. For each variety of conceptual understanding there are specific tasks that the student should be able to execute. This framework was created based on literature available, the tutor's own mathematical expertise in teaching mathematics, and dissection of transcribed sessions of inquiry-oriented classroom teaching [Rasmussen et al., 2008]. The six competencies that express relational understanding of procedures are listed in Table 1.

In this paper we try to find evidence of these six relational understanding competencies in the tutoring sessions. Table 1 illustrates attested samples of tutoring that correspond to five of the six competencies, as well as corresponding tutoring goals. This study is thus an early test of the framework.

Relational Understanding Framework	Example Dialogues Found in Transcripts	Tutoring Goals
1. Anticipate the outcome	T: OK-- if I asked you to "solve" this de, what kind of answer would you get? A number? A variable? Something else? S: A variable T: OK-- what variable would you get, do you know? S: t but in terms of y T: That is one possibility-- if you get a variable in terms of another variable, we usually call that a function. [Transcript DE6, turns 107-4 – 111-1]	Form of Solution
2. Identify the procedure	T: OK-- so you said you will solve the function above. Can you tell me in words what you will use to do this? S: This function is separable so I will separate the y and the t, and then integrate both sides [Transcript DE1, turns 123-2 – 123-3]	Separation of Variables
3. Carry out the procedure	T: OK, now, we are "separating" so we need to divide both sides by y to get the y with the dy. Can you do that for me? S: $\frac{1}{y}dy = (t+1)dt$ Is that correct? T: Yes sir!! [Transcript DE2, turns 145-2 – 147-1]	Follow Procedure
4. Know motivation for key steps	T: Why do you use a C? S: You need the c because when you take the derivative of a function, the constant becomes zero, so the integral of a function can have infinite solutions, if there is no initial condition T: Yes, that's nice. So really, would you agree that the C creates a "family of functions" varying by a constant? S: Yes, [Transcript DE1, turns 117-2 – 120-1]	+C Concept Solve for y Eliminate ln Integrate both sides
5. Verify the correctness or reasonableness	T: I hope I can add something to your list of ideas- so let me ask you, if I said is 2 a solution to x^2+5=9, can you tell me if it is correct? And how? S: Yes just plug it in and see if it works if it didn't work, the I would work the problem until I found another solution T: I agree with you. What does "it works" mean in the x^2+5=9 example, and is 2 a solution? S: I mean plugging the number two into x and doing the operations asked in the example so 2^2 =4 and 4 +5 =9 9=9 so it is a solution that works T: Absolutely- because the 2 made the statement "true" 9=9, right? S: Right [Transcript DE8, turns 117-2 – 122-1]	Solution Check
6. Interpret Graphically	Not attested in our tutoring sessions	

Table 1: Relational understanding framework with corresponding examples of tutoring

Tutoring Sessions

Nine tutoring sessions were collected. All nine of the students used in this experiment were engineering or science students enrolled at the time in the differential equations class given through the Mathematics Department at North Carolina Agricultural & Technical State University. The tutor was Dr. Karen Keene, an experienced teacher of ODEs. Communication was keyboard-to-keyboard, with tutor and student in different locations, mediated and captured by the web-delivered Wooz software [Patel et al., 2003]. In all the transcripts the student solved the same differential equation by separation of variables. During the tutoring session the DE to be solved was always visible in an equation-editing window separate from the scrolling chat window. When more equations are created and edited during the conversation they remain displayed in the equation window and we interpolate them into the transcript of the dialogue. Every part of the human-to-human tutoring sessions were saved in log files.

On average, one complete tutoring session lasted nearly one hour and one minute, with an average of 61.4 turns taken by either the student or tutor. The shortest time for one session was fifty minutes and the longest session lasted one hour and eight minutes.

Length of session	01:01:41
Average number of dialogue turns	61.4
Average learning gain	0.69
Average learning gain for course grade A,B,C students	0.83
Average learning gain for course grade D,E students	0.59

Table 2: Statistics of tutoring sessions

A pre-test and post-test were given to each student before and after completion of the tutoring session. This test consisted of two problems which the student had to solve by separation of variables. The same two questions were given for both tests (pre and post). The average learning gain was 0.69 for the nine students, where each student's learning gain was measured by (post-score – pre-score) / (1 – pre-score), and the average was calculated by (average-pre-score – average-post-score) / (1 – average pre-score). The learning gain average for the 4 students who received final grades A, B, or C in the DE class was 0.83, while the learning gain for the 5 students who received lower grades was 0.59. Table 2 summarizes the statistics for the nine tutored students.

Tutorial Goals

Here we describe the steps that the tutor executed in almost every tutoring session. From the point of view of planning a tutorial conversation, each step is a goal that must be satisfied before proceeding. These goals have mixtures of pedagogical and conversational value, for example the *greeting* goal satisfies both a conversational need (it would be infelicitous to start most conversations without any sort of greeting) and a pedagogical need (the teacher inquires about the student's level of preparedness). The main goals that we found are:

1. greeting/getting started
2. form of solution
3. separation of variables problem
4. follow procedure
5. solution check
6. problem review

Some of the goals had sub-goals that were performed to help complete the goal they correlate to. The sub-goals are:

1. solution check (with examples)
2. +c concept
3. mathematical shortcut
4. integrate both sides
5. eliminate ln
6. chain rule concept
7. separation of variables examples

With these goal and sub-goal breakdowns of each transcript, goal hierarchies (flow charts) were created as a model of the flow of the tutoring session from beginning to end. Figure 1 shows the flow chart for the nine transcripts.

Goal *greeting/getting started* was used to primarily let the student and tutor introduce themselves and gain knowledge of the student's current knowledge about the separation of variables problem, which is displayed in a separate window. The point at which the problem is displayed is noted in the transcript below. This goal began each session before any problem solving got underway. Here is an example:

tu: Hello. My name is Karen and I teach Differential Equations-- what is your name? And could you tell me what college math classes you have had before DEs, please
st: My name is _____ . I have had Pre-Calc, Calculus, and Calculus 2
tu: OK-- that sounds good- ____, have you had Calculus 3? Or will you take it?
st: No I don't have to take it
tu: Ah- so what is your major?
st: Computer Science
tu: $\frac{dy}{dt} = y(t+1)$ [The problem is displayed now]
tu: OK- so what we want to do is go back to earlier in the semester and look at the solving of an earlier differential equation. I put one in the box, using y as the dependent variable and t as the independent variable. To solve this, you will find a function y(t) that makes this true. **[DE7: 101-122]**

Goal *form of solution* addresses the first competency in the framework for relational understanding of procedures, "anticipate the outcome." The student should know that the result of solving this DE is a function or a family of functions of the form y = y(t). The solution is not a number and can possibly be a specific solution if an initial condition is given. It is not *a priori* obvious that this goal should be the first part of tutoring, conceivably it would be possible to mechanically solve the problem and ask the student what form of solution emerged. However in practice our expert tutor addresses this item of relational understanding before having the student attempt to solve the problem. Typically the first attempt to satisfy this goal consists of asking the student what form the solution will take. If the response is not adequate, an alternate pattern of sub-goals consists of *function* and *general solution*. Here is a simple example of the *form of solution* goal:

tu: Let me ask you a couple more and then we will get there-- if you solve a differential equation, what kind of answer would you get? Do you get a number? A variable, an equation, a function? Something else?
st: An equation...?
tu: Pretty close- you get something that looks like an equation but is actually a function. So to solve this, you are looking for y(t) that makes this DE true. Sometimes we just say y, but we mean y(t). **[DE2: 111-115]**

Goal *solution check* correlates to the last framework concept, "verify the correctness or reasonableness." Once the student realizes what the form of a solution looks like, the tutor can review with the student how to check that a given function is a solution to a specific differential equation problem. Checking solutions requires plugging the function into the original differential equation. Understanding *how to plug-in* is one of the sub-goals of this task. At times, simple algebraic examples are given to help the student with the concept of plugging-in to obtain an answer. An interesting feature of the *solution check* goal is that sometimes it occurs early in the dialogue before the solution is found, and sometimes after the solution is found during the *follow procedure* goal.

tu: OK-- back up a minute- to put this in perspective-- If I said is x=2 a solution to x^2+6=10, how would you know?
st: Plug in 2 for x into the equation to check and it equals 10.
st: For the previous equation the derivative of y would be e^...(t+1)
tu: EXACTLY-- and so if you put that on the left, then on the right, put in what y= for y.
tu: Check if the left and right side are the same— would they be? We could write it in if you need
st: They're the same.
tu: So the solution is right for sure--- nice job. **[DE6: 155-159]**

Goal *present separation of variables problem* addresses the "identify appropriate procedure" competency. Transcripts usually follow the form of presenting the problem and then the tutor asks the student what method, from the ones taught already in their differential equations class, would they use to solve this problem. Sometimes, the tutor even asks the student *why is separation of variables* the method in which they would use to solve. Another sub-goal helps the student with understanding *how to choose* if the separation of variable technique is applicable to the problem at hand. The hint for this sub-goal is to get one side as the product of a function of one variable multiplied by a function of another, then divided by the function of the dependent variable. It is important to note that this goal assumes that the student has already learned about separation of variables from their DE class, this part of the tutoring dialogue focuses more on making sure that the student understands why this is an appropriate technique for the problem at hand. The following example is illustrative:

tu: Do you remember the technique of separation of variables?
st: Yes get the y terms together and t terms together
tu: Sounds pretty good.
tu: Before we do that, can you tell if this problem can be solved with separation of variables? If so, how do you know?
st: I think it can, because u need to have your y term with dy by dividing and multiply both sides by dt to get it with the t+1 function
tu: Sounds good **[DE5: 125-129]**

Goal *follow procedure* correlates to both the third competency of the framework, "carry out the procedure" and the fourth "know motivation for key steps." The tutor gives hints as to where to start if the student is stuck in the beginning. Within this goal, student and tutor process of separating the two variables, *integrating both sides correctly* and then solving algebraically for their function of y (*solve for y)*. Included in this goal, alternating with procedural steps, is verification as to why the student is able to separate the dy/dt when separating (*mathematical shortcut*). *Correcting algebra mistakes* also take place as a sub-goal of this procedure. These sub-goals assist the student with obtaining the final function that correctly solves the initial differential equation. The "know motivation for key steps" competency is addressed by sub-goals such as *+c concept* after integration. The tutor also typically interpolates "Why…" questions with the procedure-execution steps to help address the "know motivation" competency. Here is an example of part of tutoring the procedure.

tu: So can you finish the problem I started by integrating both sides? You can write it in the box.

st: $\frac{y^3}{3} = (t^4 / 4) + t$

tu: Nice- your integration looks good.
tu: However, you forgot your +C.
tu: Did you learn to add a C when you integrated in Calc II? Go ahead and add that in

st: $\frac{y^3}{3} = (t^4 / 4) + t + C$

st: Yes **[DE9: 123-126]**

Goal *Review/Identify Method Type* simply reviews the problem and checks the student's understanding of when to use the separation of variables method. In this goal, the tutor reassures the student of the form of the DE in order for separation of variables to be applicable. The goal sometimes re-states the steps they performed throughout their tutoring session to reinforce what the student just learned.

Discussion and Conclusions

The fact that the expert tutor addresses each of the high-level goals described here in almost every session is evidence that tutoring this problem is schematic. It should be possible to computerize the goal structure, to follow the conversation with new students, and to generate new tutoring mechanically. It is emphatically not the case that each time this problem is tutored the conversation goes in a different direction.

With regard to the *form of solution* goal, understanding that the solution to a differential equation is a family of functions, rather than a number or a single function, addresses a common expectation from earlier mathematics classes in the student's career, viz: results are usually numbers or equations [Rasmussen, 2001]. This may be one reason why many students have difficulties when it comes to a differential equations course. Once a student conceptualizes solutions as functions rather than numbers, students become more successful in solving differential equations problems [Rasmussen, 2001]. We also notice the striking similarity with algebra tutoring [Kim and Glass, 2004], where the first substantive goal was to get the student to express the operation to be performed. In both DE and algebra tutoring, the task at hand is being framed in the student's mind.

With regard to the *solution check* goal, we note that it can either precede or follow finding the solution, since the competency is to know *how* to check a solution. We have examples where the tutor tried this goal twice in the same dialogue, both before and after finding the actual solution.

The tutor chooses in these dialogues to intertwine both solving the equation and ensuring the student knows the motivation for key steps. This contrasts with the other competencies, which are each addressed in separate conversational segments.

Student answers to specific questions that are not what the tutor desired often prompted a variety of response strategies to assist the student. Zhou et al. [1999] categorized student answers and the subsequent tutor approaches for the CIRCSIM-Tutor project. We observed a number of similar behaviors in this study, particularly retrying after partial answers. The retry after a partial answer technique involves the tutor acknowledging the partially correct part of the answer, and then focusing on the missing part. In many other cases the tutor will just inform or give hints to the student of what is missing, and then ask the student about it for reassurance.

As illustrated in Table 1 above, comparing the framework for assessing relational understanding of procedures to tutor behavior shows a strong correlation. The sequence of tutoring goals addresses the first five of the competencies that students should acquire in order to exhibit relational knowledge. The sixth, a skill involving characterizing equations from graphical representations such as vector fields, was not a part of our tutor's agenda. The Wooz tutoring interface provided poor support for the needed graphical illustration. For example, an assessment for this competency involves showing the student a slope field and asking questions about it. The goal hierarchy, embodying much of the list of competencies, served as the high-level plan for the entire tutoring session. In the spirit of the research plan laid out by Fox: "[T]he study of human-human instructional dialogue is a critical step in the further development of intelligent tutoring systems, in that such research will provide the theoretical foundation on which working systems can be built" [Fox, 1991], this study extends that plan.

Acknowledgements

We would like to thank a group of hard-working NC A&T students whom analyzed transcripts and came up with their own goal hierarchies before comparing with the framework provided by the expert tutor. These students include Jason Davidson and Madhuri Yalamuru.

This work is supported by the Reese Program of the National Science Foundation, under awards 0634049 to Valparaiso University and 0633953 to North Carolina A&T State University. The content does not reflect the position or policy of the government and no official endorsement should be inferred.

References

Evens, Martha and Joel Michael. 2006. *One-on-one Tutoring by Humans and Computers.* Lawrence Erlbaum Associates, Inc: Mahwah, New Jersey.

Fox, Barbara A. 1991. Cognitive and Interactional Aspects of Correction in Tutoring. In Goodyear, Peter, ed. *Teaching Knowledge and Intelligent Tutoring,* Bristol, UK: Intellect Books, pages 149-172.

Freedman, Reva. 2000. Plan-Based Dialogue Management in a Physics Tutor: *Proceedings of the 6th Applied Natural Language Processing Conference, Seattle*, pp. 52--59.

Glass, Michael, Jung Hee Kim, Karen Allen and Kathy Cousins-Cooper. 2007. Towards Wooz-2: Supporting Tutorial Dialogue for Conceptual Understanding of Differential Equations. *Eighteenth Midwest AI and Cognitive Science Conference, Chicago,* pp. 105--110.

Graesser, A. C., Olney, A., Haynes, B. C., and Chipman, P. (2005). AutoTutor: A cognitive system that simulates a tutor that facilitates learning through mixed-initiative dialogue. In C. Forsythe, M. L. Bernard, and T. E. Goldsmith, Eds., *Cognitive systems: Human cognitive models in systems design.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Grosz, Barbara J. and Candace L. Sidner. 1986. Attention, Intentions, and the Structure of Discourse. *Computational Linguistics, 12 (3),* pp. 175--204.

Jordan, Pamela, M. Ringenberg, and B. Hall. 2006. Rapidly developing dialogue systems that support learning studies. In *Proceedings of ITS06 Workshop on Teaching with Robots, Agents, and NLP*.

Keene, Karen. 2009. Private communication.

Kim, Jung Hee, Reva Freedman, Michael Glass and Martha W. Evens. 2006. Annotation of Tutorial Goals for Natural Language Generation. *Discourse Processes, 42(1),* pp. 37--74.

Kim, Jung Hee, and Michael Glass. 2004. Evaluating Dialogue Schemata with the Wizard of Oz Computer-Assisted Algebra Tutor. *Intelligent Tutoring Systems: 7th International Conference*, *ITS 2004 Maceio, Brazil*, Springer, Lecture Notes in Computer Science no. 3220, pp. 358--367.

Patel, Niraj, Michael Glass, and Jung Hee Kim. 2003. Data Collection Applications for the NC A&T State University Algebra Tutoring Dialogue (Wooz-tutor) Project, *Proceedings of the Fourteenth Midwest Artificial Intelligence and Cognitive Science Conference*, Cincinnati, OH.

Rasmussen, Chris L. 2001. New directions in differential equations: A framework for interpreting students' understandings and difficulties. *Journal of Mathematical Behavior 20 (2001)* pp. 55-87.

Rasmussen, C., and Kwon, O. (2007). An inquiry oriented approach to undergraduate mathematics. *Journal of Mathematical Behavior, 26,* pp. 189-194.

Rasmussen, C., O. Kwon, and K. Marrongelle. 2008. A Framework for Interpreting Inquiry-Oriented Teaching. *Conference on Research in Undergraduate Mathematics Education, Mission Valley, CA.*

Zhou, Yujian, Reva Freedman, Michael Glass, Joel A. Michael, Allen A. Rovick, Martha W. Evens. What Should the Tutor Do When the Student Cannot Answer a Question? *Proceedings of the Twelfth International Florida AI Research Society Conference (FLAIRS-99), Orlando, FL, 1999* , AAAI Press, pp. 187-191.

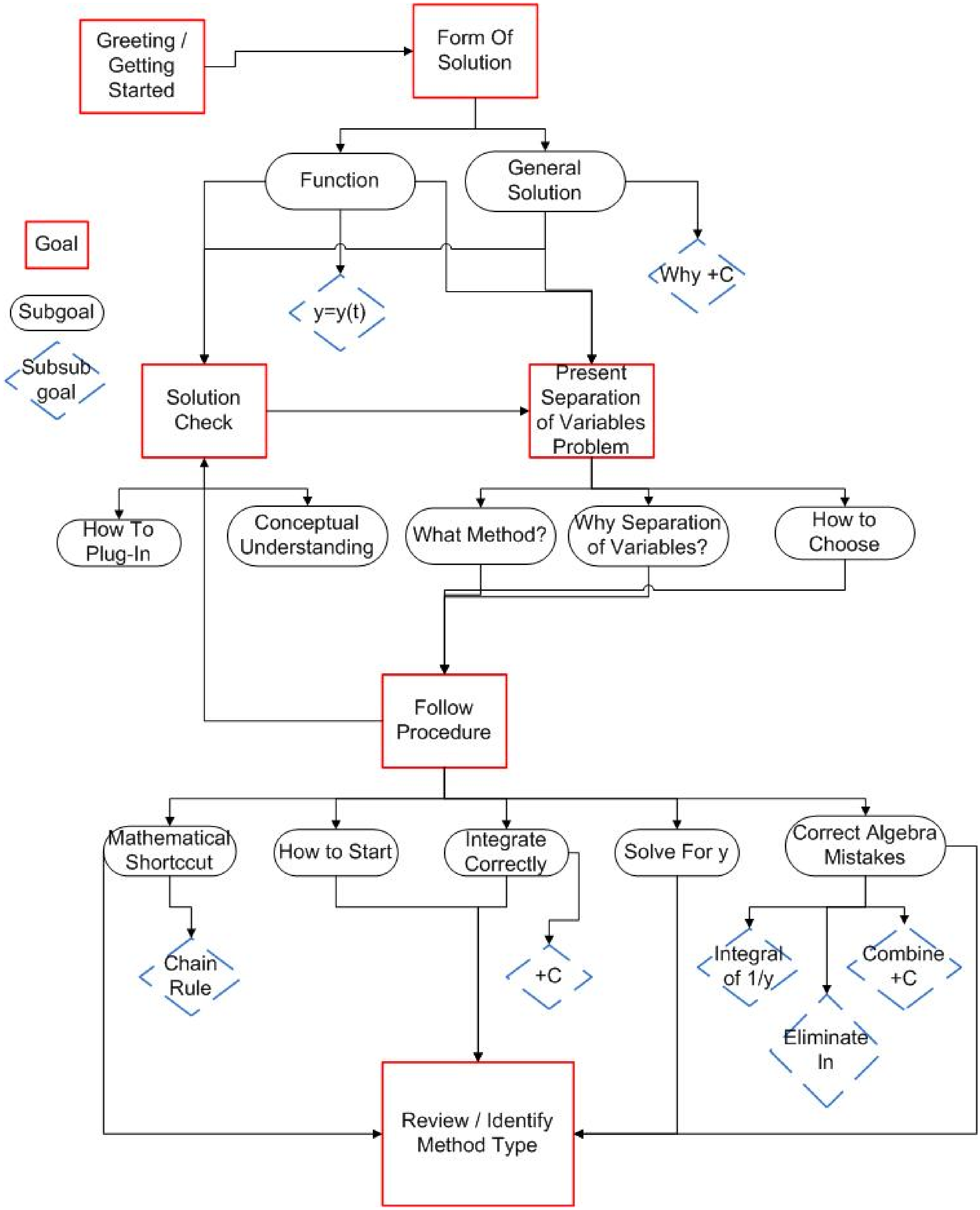

Figure 1: Goal Hierarchy / Flow Chart