6. COLLISIONS AND CONSERVATION LAWS

OVERVIEW:

This week you will use the World-in-Motion software to analyze the video of a collision between two pucks on an air table. During the analysis you will test to see whether momentum and energy are conserved and will consider the type of collision that occurred: elastic, inelastic, or perfectly inelastic.

LEARNING OBJECTIVES:

- Calculate linear momentum of an object
- Test to see whether the momentum of a system of objects is conserved in a collision
- Calculate the kinetic energy of the system before and after the collision
- Understand the difference between elastic, inelastic, and perfectly inelastic collisions

PROCEDURE:

The general purpose of this laboratory exercise is to test the hypotheses that linear momentum and the total mechanical energy are conserved in a two-particle collision in an isolated system. We define the total linear momentum

$$\vec{p}_{total} = \sum \vec{p} = \sum m\vec{v} \tag{1}$$

Q: If two or more objects interact by internal forces only – such as in a collision – are the total momentum and mechanical energy the same *before* and *after* the collision? Explain the conditions under which momentum and energy should be conserved in a collision.

We will be using a two-particle system – two pucks on an air table. This system is essentially isolated because the air table very effectively negates the force of gravity on the pucks, and because the thin film of air on which the pucks ride very nearly eliminates the frictional forces between the pucks and the air table. The motion of interest occurs within the horizontal plane.

1. Recording the Collision

You will use a video-camera technique (similar to that used in the projectile motion laboratory exercise) to record the position of the pucks at known time intervals. The lab instructor will help you obtain a digital video file of the collision. Typical video has 30 frames per second, but yours should be saved with a rate of 15 frames per second. While you are recording your video clip, observe the apparatus and think about the various factors that might contribute to the experimental error (especially possible systematic errors) in the position measurements.

Use the World-in-Motion program (review Appendix F) to locate the positions of the pucks at the times when the camera shutter was open. Don't forget to carefully carry out the calibration procedure in World-in-Motion to get accurate results.

2. Calculating momentum and energy

After copying and pasting the World-in-Motion position data into a spreadsheet, make a single plot of $y(t)$ vs $x(t)$ for both pucks before and after the collision. This plot will provide you with an overview of the paths of the pucks. Next, determine the total momentum of the two-particle system before and after the collision. To calculate $\vec{p}_{total}$ you will need to determine the momentum of *each puck*, first before and then after the collision. Then add the momenta of the two pucks before the collision (and after). Since $\vec{p}_{total}$ is a vector, you will need to compute and then add its *components* along the coordinate directions, as follows:

$$p_{x,total} = m_1 v_{1,x} + m_2 v_{2,x} \quad \text{and} \quad p_{y,total} = m_1 v_{1,y} + m_2 v_{2,y} . \tag{2}$$

When you are done you will have values for $p_{x_{total}}$ and $p_{y_{total}}$ for the system before and after the collision. But how do you calculate the momentum components? First you need to obtain the velocity components of each puck before as well as after the collision.

To determine the x and y components of the puck velocities, first plot x vs. t for one of the pucks before the collision. **What kind of behavior does this graph show? Is this what you expected?**

Analyze this graph to determine the v_X for this puck. In order to find the information you need for the comparison of momenta and energy, you will need to perform this kind of analysis 8 times. **Do you see why you'll need to perform the analysis 8 times?** In the end, plot all the $x(t)$ data sets on one graph and all the $y(t)$ data sets on another graph.

Once you have done the necessary calculations, enter the velocity components of each puck (before and after the collision) and their errors in the spreadsheet provided for you. This spreadsheet is organized in a way that will make the calculation and comparison of momenta and energy easy to carry out. Then calculate the *components* of the total momentum of the system before and after the collision. Also calculate the total kinetic energy of the system before and after the collision

$$K_{i,total} = \frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 \tag{3}$$

(Note: since the gravitational potential energy can be set to zero at the level of the air table, your value for K is the total mechanical energy E.)

3. Final analysis

Now find the uncertainty associated with each of the momentum components you calculated above. You will need to use the uncertainty values in the velocity components as well as the uncertainty in the measurements of the puck masses. As you have learned in previous laboratory exercises, this step is crucial if you want to make any meaningful comparisons. In the spreadsheet provided for you, one of these error formulas is already entered. Examine the formula and make sure you understand where it comes from.

When you are finished with these calculations, compare the components of the total momentum of the system before and after the collision. In this experiment, the best way to do this is to calculate the change in the components of the total momentum of the system that occurred during the collision. This means you will calculate the following quantities:

$$\Delta p_{x,total} = p_{f,x,total} - p_{i,x,total} \quad \text{and} \quad \Delta p_{y,total} = p_{f,y,total} - p_{i,y,total} \tag{4}$$

You will also need to calculate the errors in the momentum changes. If the total momentum of the system is conserved in the collision, what values do you expect to get for $\Delta p_{x,total}$ and $\Delta p_{y,total}$? Why?

What conclusion about total momentum can you draw from your data? (Be sure and make use of your error analysis work above.)

Next, calculate the uncertainties in the energy, both before and after the collision. Again, one of these calculations is already entered as an example to you. Compare the energy before and after collision, using the appropriate error calculations. Just as in the case of the momentum calculations, the best way to do this is to calculate the change in the energy of the system during the collision (along with its error):

$$\Delta K_{total} = K_{f,total} - K_{i,total} \tag{5}$$

What conclusion can you draw regarding the mechanical energy from your data? Can you determine what kind of collision occurred? Explain.

TURN IT IN:

1. Your spreadsheet, containing the measured position data, your linear regression analysis, and your calculations of the system's momentum and energy before and after the collision.
2. Your two position vs. time graphs (one with all the *x* vs. *t* data sets and another with all the *y* vs. *t* data sets), with the fit lines for each data set.
3. Your completed report form.