7. MEASURING THE MOMENT OF INERTIA OF A ROTATING HOOP

Read Appendix G before coming to the laboratory

OVERVIEW:

In this week's laboratory exercise you will use a rotating apparatus to measure the moment of inertia *I* for a thick-walled cylindrical hoop and compare your result to the value expected for an ideal, uniform object with this shape. You will also examine the influence of frictional forces on the value measured for *I* and, if necessary, calculate a correction for this effect.

LEARNING OBJECTIVES:

- Understand the relationship between the tangential velocity and angular velocity for a rotating object.
- Understand the meaning of the moment of inertia of a solid object.
- Apply Newton's 2^{nd} Law in its rotational form to calculate the moment of inertia *I*.
- Learn how to correct (approximately) for the frictional forces that act on the rotating hoop.
- Apply all of the good laboratory techniques developed this semester, including error analysis.

APPARATUS:

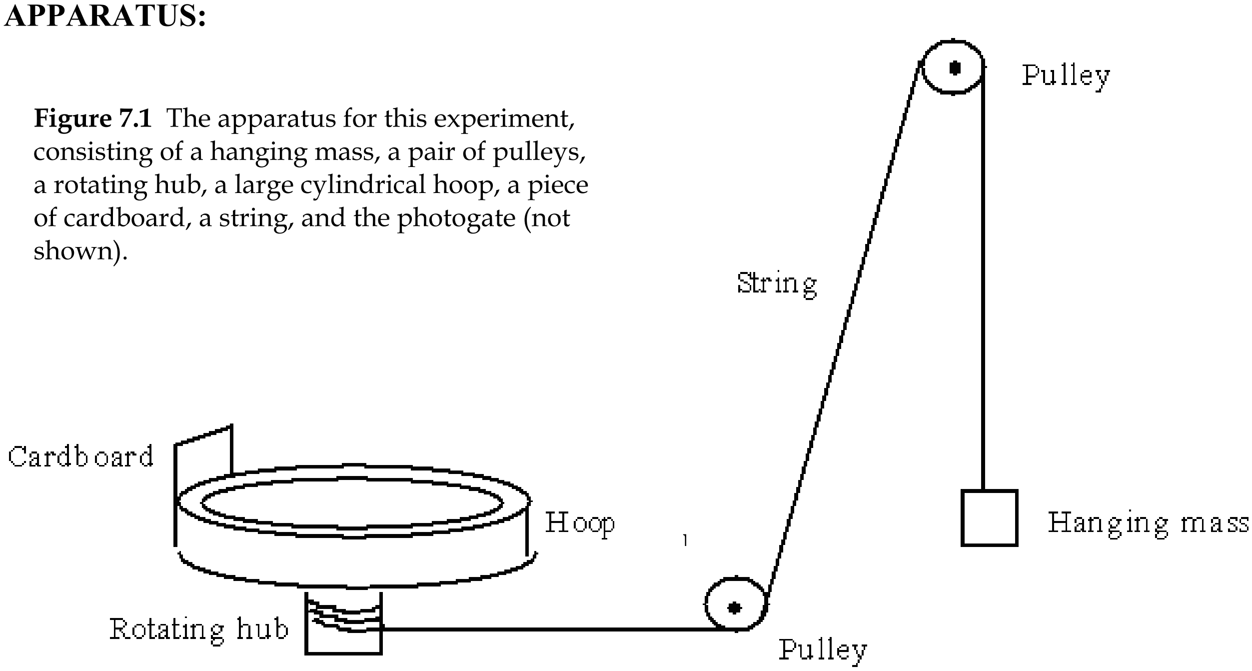

Figure 7.1 The apparatus for this experiment, consisting of a hanging mass, a pair of pulleys, a rotating hub, a large cylindrical hoop, a piece of cardboard, a string, and the photogate (not shown).

The apparatus used for this laboratory exercise is shown above. When the hanging mass *m* falls freely, it creates a tension, *T*, in the string. The string is wound around the hub of the rotating hoop. The tension of the string causes the hub and the massive hoop to rotate. When the hoop rotates, the precision timer measures the *time* it takes for the piece of cardboard (taped on to the outer surface of the hoop) to cross the photogate during each revolution (see Appendix G). This measured time can be used to determine the linear velocity of the outer surface of the hoop. From this, you can calculate the angular velocity of the hoop. Knowing the angular velocity of the hoop for each revolution, you can determine the angular acceleration of the rotating hoop, the torque acting on the hoop, and thereby the moment of inertia of the hoop.

THEORY:

The torque that causes the hoop (and the supporting hub) to accelerate is due to the force of tension applied by the string to the hub. However, another torque is applied by frictional forces in the bearings of the hub. So, we can write Newton's Second Law for rotational motion:

$$\sum \tau = \tau_{string} + \tau_{fric} = I\alpha \tag{1}$$

where the frictional torque is negative. This can be used to solve for the moment of inertia of the system:

$$I = \frac{\tau_{string} + \tau_{fric}}{\alpha} \tag{2}$$

To determine *I* for this system, therefore, you need three quantities:

- The torque produced by the string on the hub and hoop,
- The frictional torque acting on the hub and hoop,
- The angular acceleration of the system.

If the frictional torque is small compared to the torque produced by the string, perhaps it can be neglected. You will see.

In using your measurements to determine the moment of inertia of the hoop, the following physics principles and formulae (which you have already learned in class) will be used:

- Kinematic relation between angular velocity and displacement

  $$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

- Relationships between linear and angular velocities/accelerations $v = R\omega$, $a = R\alpha$
- Definition of torque $\vec{\tau} = \vec{r} \times \vec{F}$
- Newton's 2nd Law for rotational motion $\sum \tau = I\alpha$
- Moment of inertia of a thick-walled cylinder about its axis, $I = \frac{1}{2} M(R_1^2 + R_2^2)$

  where *M* is the mass and R_1 and R_2 are the inner and outer radii of the cylinder.

First you will use the measurements from the photogate timer to find the angular acceleration of the hoop and hub. To determine the torque produced by the string, you need to determine the tension in the string while it is wound around the hub. You can use a free-body diagram of the hanging mass, correctly apply Newton's 2^{nd} Law, and then use the rotational data to find the linear acceleration of the hanging mass and determine the tension force.

If you can use this method to determine the tension, you need only to use the definition of torque to find the torque produced by the string on the hub. With the torque and angular acceleration, it is simple to find the moment of inertia of the system.

PROCEDURE :

I. INITIAL SETUP

Since you will be finding the experimental moment of inertia of the hoop, you will want to compare it to the expected value from the formula above for an ideal thick-walled cylinder. Make the necessary measurements to determine I. (You will calculate its value later.) Pay attention to the errors in your measurements!

Cut a piece of cardboard (~ 3 cm, perhaps from a stiff index card) for blocking the light of the photogate. Measure the length of the cardboard that will obstruct the light in the photogate. Tape the cardboard to the outer surface of the hoop as shown in Figure 7.1. Later on you will determine the linear velocity of the outer surface of the hoop by measuring the time it takes for this piece of cardboard to pass through the photogate.

Set up two pulleys as shown in the figure. Use the foot screws to level the rotation apparatus on the tabletop. Be sure that the string between the hub and the first pulley is parallel to the tabletop. **Why must the string be level? The answer to this question should be familiar to you!**

One end of the string should have a loop that can go around the pin on the hub of the apparatus. DO NOT TIE A TIGHT KNOT. Use string to wrap around hub ~10 times and reach the overhead pulley (~3.5 m). Cut string if needed.

Hang a small mass (~ 150 - 200 g) at the end of the string. Release the mass and observe the motion, letting the mass strike the floor. The string should come off the hub **just before the mass hits the floor** and the hub should keep rotating, slowed by the frictional forces in the hub. If the string doesn't come off the hub before the mass hits the floor, the string may wrap around the hub or even go inside the hub and introduce some unwanted frictional forces. Adjust the length of the string as necessary.

II. COLLECTING THE PHOTOGATE TIMER DATA

Open the file on your computer desktop entitled "**Rotating_Ring.cmbl**." This file will start **Logger-Pro** with all the correct options pre-selected. Practice using the precision timer software. If used in **Gate Timing** mode, the timer measures the time interval during which the light is blocked by the piece of cardboard. This time interval and the width of the piece of paper can be used to determine the instantaneous linear speed of the outer surface of the hoop.

When you are ready to take timing measurements, wrap the string around the hub, release the mass from near the overhead pulley and collect time data (one reading per revolution) **both before and after the hanging mass hits the floor**. After it hits the floor, continue collecting measurements until the hoop slows nearly to rest.

The first part of the time data will be used to determine the angular acceleration of the hoop and the tension in the string. As the hoop "coasts" after the string has unwound completely, only the frictional forces act on it to slow its rotation. The second part of the time data will be used to determine the angular acceleration (actually a deceleration) caused by the frictional forces.

Copy the precision timer data to a spreadsheet. There will be 3 columns of data: the first column is the time of each event, the second is the revolution number, and the third column is the time Δt in seconds during which the photogate was blocked by the piece of paper. Show your data to your instructor and get his/her approval before proceeding.

III. CALCULATING THE ANGULAR ACCELERATION

For each datum point, calculate in the spreadsheet:

(a) the velocity of the outer surface of the hoop
(b) the angular velocity of the hoop
(c) the total angular displacement theta (choose the first point to be theta = zero)

Plot a graph of ω^2 vs. theta. Use this graph to determine:

(a) the *average angular acceleration produced by the string (and friction)*, and

(b) the *average angular acceleration due to friction alone* (watch out for algebraic signs).

Determine the errors in the above quantities. Is the influence of friction negligible for your apparatus?

IV. CALCULATING TORQUE AND THE MOMENT OF INERTIA OF THE HOOP

The torque produced by the string can be determined if we know the tension of the string. This can be determined by applying Newton's 2nd Law to the motion of the hanging mass.

Refer to the free-body diagram you drew in the pre-lab for the hanging mass. From this, you were able to write down Newton's 2nd Law. If we know the acceleration $\boldsymbol{a}$ in this equation, we can determine the tension T. (A reasonable value to use for the earth's gravitational acceleration at our latitude and longitude is $g = 9.803 \pm 0.001$ m/s^2.) Note that the acceleration of the outer surface of the *hub* is the same as the acceleration of the falling mass m. Use the angular acceleration value you have calculated to determine the acceleration of the falling mass.

Use these results to determine T, then find the torque τ_{string} produced by the string (along with their errors!).

Now, ignoring for the moment the influence of friction, use the information you have to calculate the moment of inertia of the rotating system, along with its error. Then calculate the value expected using the formula given earlier, along with its error. Compare these values and comment on the comparison.

V. CORRECTING FOR THE EFFECTS OF FRICTION

Since we do not know the precise nature of the frictional forces acting inside the bearings in the hub, we cannot easily calculate the frictional torque τ_{fric}. However, we can still correct for the influence of friction by using Newton's 2nd Law in rotational form:

$$\tau_{fric} = I\alpha_{fric} \tag{3}$$

Now substitute this into equation (1) above, and solve for *I* in the resulting equation. This gives an expression for the moment of inertia that includes an approximate correction for frictional torques in the bearings. Calculate your "corrected" moment of inertia and once again compare to the expected value.

TURN IT IN:

1. Your completed lab report form.
1. A copy of your spreadsheet including the raw data, your calculations, and error analysis.
2. A copy of the graph of ω^2 vs. θ, with the appropriate fit line(s).