2. PROJECTILE MOTION

Read Appendix F before coming to the laboratory

OVERVIEW:

You will digitally record the flight of a ball that is in free fall after being launched from a spring-loaded device. With the help of some computer analysis software, you will mark the position of the ball in the series of recorded video frames and use the resulting position versus time data to determine the behavior of the ball's motion in both the horizontal and vertical directions. Ultimately, you'll be able to derive a value for g, the magnitude of the acceleration due to gravity, from your data.

LEARNING OBJECTIVES:

- Analyze motion in video files using computer software
- Understand the horizontal and vertical motions of a ball in free fall
- Be able to perform a linear regression to fit your data
- Develop an understanding of uncertainty and sources of error

PROCEDURE:

1. RECORDING THE MOTION

To study the motion of an object in two-dimensions, we will use a video camera with a fast shutter speed. The resulting series of two-dimensional images provides no direct information about motion perpendicular to the plane of the video frame. Therefore, if the plane containing the motion of the object is not exactly parallel to the plane of the video camera detector, you will not get a completely reliable representation of the motion. This should say something about the care with which you need to set up your apparatus and observe the motion of the ball.

Since a standard video camera records images at a rate of 30 per second, it can be used to record the instantaneous (x, y) location of the object at successive times, as shown in Fig. 2.1. For this reason, we will use the video camera to record the (x, y) position of the ball at specific instants of time (t). The video images will be digitized and stored in a computer file that you can view and analyze at your lab station. From these data, you can study the motion of the ball in the horizontal direction (x) and in the vertical direction (y), each as a function of time. Moreover, from your study of motion in the y-direction, you should be able to obtain a measurement of the acceleration due to the force of gravity at this location on the earth's surface.

With the help of the lab instructor, each group should use the spring-loaded launcher to launch the ball and obtain a video file of the position of the ball as it undergoes "free-fall" motion. You will analyze this video file with the World-In-Motion software to measure the position of the ball as a function of time.

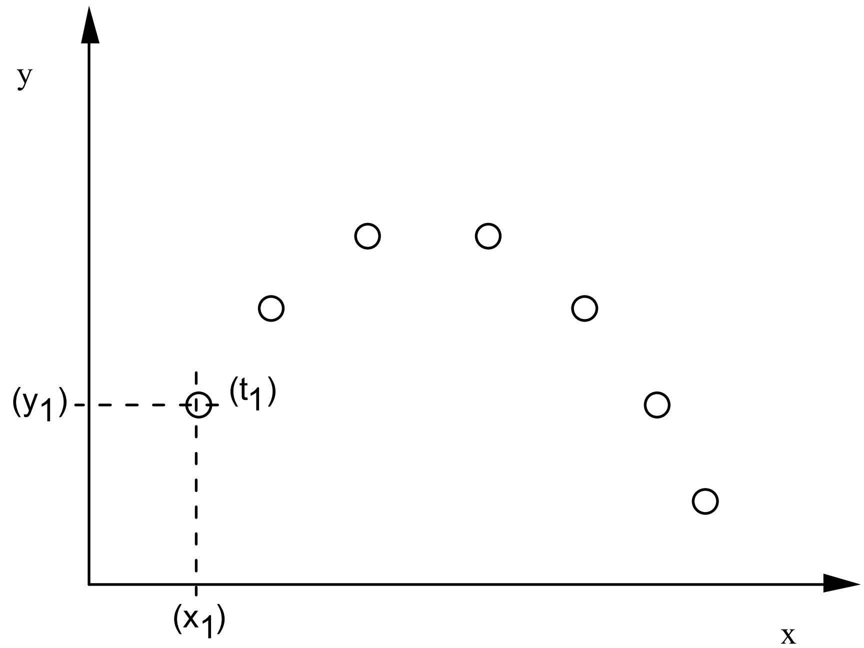

Figure 2.1. This illustration shows the superimposed images of an object moving in two dimensions (*x*, *y*) seen by a video camera for each video frame. From a figure like this, the (*x*, *y*) positions of the object at successive time intervals (*t*) can be determined.

2. OBTAINING $x(t)$ AND $y(t)$

Now your task is to find the (*x*, *y*) coordinates for each image of the ball in your video clip. These measurements are done using the World-in-Motion program by clicking on the position of the ball in each video frame. Be sure to limit the video to only include frames where the ball is visible. Refer to appendix F in your lab manual to find instructions on how to use this program. Make special note of the procedures discussed below.

Calibration

Because you will need values of (*x*) and (*y*) which are in meters, you will have to **calibrate** the distances on your computer screen. This calibration is done with the Video Scale function in the World-in-Motion program. You need only calibrate in the horizontal or vertical direction, not both. Measure the length between two widely spaced, but clearly identifiable points on the meter stick that is off to the side in the video. In your spreadsheet, record your observations about the various factors that will cause experimental error in your position measurements.

After you complete this calibration procedure, use the software to measure the (*x*) and the (*y*) coordinates of the two points you used to do the calibration, as well as at least 5 other observable points along the meter stick. Record these values in your spreadsheet, and compare the distances between these points as given by the software to known values in the lab. This calibration "check" will confirm your calibration and will provide a measure of the uncertainty in the measurement of points in the picture.

This information will also allow you to check whether the picture appears to have the *y*-axis vertical. **Can you see how to verify the vertical alignment?** If either of these checks give evidence of a problem in the calibration, you can repeat the calibration and/or repeat the data collection altogether.

Assessing Random Error

It is also useful to check to see the finest increment in x-and y-positions that World-in-Motion is capable of distinguishing. Examine the X-Coordinate box at the bottom of the screen and move the mouse *just slightly* in the horizontal direction. What is the minimum increment in x-position? Repeat this in the y-direction. One-half of these values is a good approximation for the random error inherent in each position measurement.

Collect the *x*, *y* Data

Advance through the frames (you can use the right mouse button to advance a frame) while clicking on the ball in each frame (using the left mouse button). When collecting the data, pay special attention to the care with which you are positioning the cursor on the images of the ball. When you are done, you will have a data set containing the (x, y) coordinates of the ball at each point in its motion. As mentioned earlier, in your spreadsheet record your observations about additional factors that will contribute to experimental uncertainty in your measurements.

ANALYSIS:

1. Import Data to Excel

Copy and paste your (t, x, y) data from the World-in-Motion program into a spreadsheet. Also, enter any other measurements you have made. Don't forget to properly label your data columns and to include units! The four rightmost columns labeled with R, D, etc. contain extra information not needed here and should be deleted. To make sure that your data have been faithfully recorded and copied, ***plot a graph of the y coordinate vs. the x coordinate***. Does it have the expected shape? (You do NOT need to print this graph.)

Now plot x vs. t and y vs. t together on the same graph. **What qualitative conclusions can you draw from these plots? Does what you see for each of these plots agree with what you expect?** You should explain your reasoning in terms of the physics involved. If you are uncertain whether you understand these plots, discuss this with your lab partner(s) and your instructor before proceeding.

2. Determining v_x AND v_y

Next, use the spreadsheet to ***calculate the velocity components*** of the ball along the x and y directions at each time. After you do this, you will have two new columns in your spreadsheet, one for v_x and one for v_y.

To calculate these velocity components using Excel, code the following velocity calculations into your spreadsheet:

Time	Position	v_x
t_1	x_1	
t_2	x_2	$v_2 = (x_3 - x_1)/(t_3 - t_1)$
t_3	x_3	$v_3 = (x_4 - x_2)/(t_4 - t_2)$
t_4	x_4	*etc.*

At time t_1 the glider is at position x_1. To estimate the instantaneous slope at t_2, use one point on either side of t_2 to compute the approximate slope at t_2. (See the notations above.) Perform this calculation on the spreadsheet for all the appropriate data. **Why will there be no estimated value of the slope (velocity) for the first and last of [*t*, *x*] points in the list?** Do this calculation in your spreadsheet (creating a new column properly labeled and with units) for v_x and v_y. Be sure *to include the units*!

Note: These calculations give the *slope of the chord* drawn between (t_1, x_1) and (t_3, x_3), the two points chosen, as the *estimate of the instantaneous velocity* at the time (t_2). Recall from calculus: the instantaneous velocity $\frac{dx}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right)$, i.e., the slope of the chord approaches the slope of the tangent as the time interval gets smaller and smaller. Therefore, experimentally, you can better estimate, or approximate, the instantaneous velocity at the time (t_2) by decreasing the time interval between (t_1) and (t_3). However, there are practical limits to this procedure.

Now, ***make a new graph, containing two plots--one for each component of the velocity versus time (i.e., v_x vs t and v_y vs. t).*** (Note, by v_x vs t, we mean that the dependent variable, v_x, should appear on the vertical-axis while the independent variable, t, should appear on the horizontal-axis.) **What does the slope of a graph of velocity versus time represent? What numerical values would you expect the slopes of your graphs to have?**

3. Regression Analysis

If either or both of your plots of the velocity components could be reasonably well approximated by a straight line, perform a linear regression analysis on the data as you did in the first lab exercise (also see Appendix D). For this exercise, you will be doing a linear regression on a velocity component versus time, for example, $v_x(t)$ vs. t. Thus, the equation for your line (for the v_x data) will be

$$v_x(t) = a_x t + v_{x0} \tag{1}$$

with a similar one for the v_y data. Not surprisingly, because these are physical quantities, they have units! It should be clear from equation (1) what the units of the slope and intercept are.

If the plots are not consistent with a straight line, do not perform a linear regression. Instead, discuss your results with your lab partner(s) and your instructor before proceeding. After performing linear regressions and adding the best-fit lines, ***print out copies of this graph*** (with appropriate labels, titles, and figure captions).

From the regression results, you should

(a) be able to obtain a value for the acceleration of the ball in the y-direction, and
(b) be able to determine its acceleration in the x-direction.

How do your values and the associated uncertainties compare with the expected values? Is there any evidence from your measurements (considering the errors) that air friction has affected the motion of the projectile?

4. Ideas for Further Work (in the second week of lab)

In order to get the maximum possible credit for this lab experiment, you must extend your work in some way. The following ideas can guide you. Discuss other ideas you have with your instructor.

a. Starting from the original video clip, repeat the entire calibration and data collection and analysis in order to test how reproducible your results are with this equipment and your analysis technique.

b. Conduct the experiment with a different projectile (a ping-pong ball, cotton ball, crumpled ball of paper, etc.) to test for possible influence of air resistance.

c. Repeat the experiment by compressing the spring in the ball launcher a bit more to generate higher speeds for the ball—will effects of air resistance be more noticeable?

TURN IT IN:

1. Your completed laboratory report form.
2. Your Excel spreadsheet, containing the experimental data, your calibration check data, your calculated velocity components, your regression results, and relevant comments
3. Your graphs of x and y vs. t, v_x vs. t and v_y vs. t.