Direct Current Electricity

Equivalent Units

A	= W/V	= N/T·m	
C	= J/V	= N·m/V	
F	= C/V	= C^2/J	= C^2/N·M
H	= V·s/A	= T·m^2/A	
J	= N·m	= V·C	= C^2/F
N	= J/m	= V·C/m	
T	= N·s/C·m	= N/A·m	
V	= W/A	= C/F	= J/C
W	= J/s	= V·A	= V^2/Ω
Wb	= V·s	= H·A	= T·m^2

Basic Complex Algebra

Review FERM Ch. 43 and Mathematics Lesson 1.

Direct Current Electricity

Electrostatics

Charges

1 Coulomb (C) = charge on 6.24×10^{18} electrons
Charge on 1 e^- = 1.6022×10^{-19} C (the inverse of 1 Coulomb)

Force on Charged Object

- General case: $\mathbf{F} = Q\mathbf{E}$ *43.17*
- Specific for 2 point charges: $\mathbf{F}_2 = Q_2\mathbf{E}_1 = \frac{Q_1 Q_2}{4\pi\epsilon r^2}\mathbf{a}$ *43.18*
  - **a** is a unit vector pointing from point charge 1 to point charge 2.

Electrostatics

Permittivity

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{\phi_{\text{actual}}}{\phi_{\text{vacuum}}}$$

$$\varepsilon = \varepsilon_r \varepsilon_0$$

NOTE: On the FE exam, assume the permittivity is $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m unless another value is provided.

Electrostatics

Electric Field Intensity

- Due to a point charge Q_1: $\mathbf{E} = \frac{Q_1}{4\pi\epsilon r^2}\mathbf{a}$ 43.20
- For a line charge ρ_L: $\mathbf{E}_L = \frac{\rho_L}{2\pi\epsilon r}\mathbf{a}$ 43.21
  - **a** is a unit vector normal to the line.
- For a sheet charge ρ_s: $\mathbf{E}_S = \frac{\rho_S}{2\epsilon}\mathbf{a}$ 43.22
  - **a** is a unit vector normal to the sheet charge.

Electrostatics

Example (FEIM):

A point charge of 0.001 C is placed 10 m from a sheet charge of –0.001 C/m^2, and a 10 m diameter sphere of charge 0.001 C is placed half-way in between on a straight line, all in a vacuum. What is the force on the point charge?

$$\mathbf{F} = \mathbf{F}_{\text{sheet}} + \mathbf{F}_{\text{sphere}} = \mathbf{Q}_{\text{point}}\left(\mathbf{E}_{\text{sheet}} + \mathbf{E}_{\text{sphere}}\right)$$

$$F = Q_{\text{point}}\left(\frac{\rho_{\text{sheet}}}{2\varepsilon} + \frac{Q_{\text{sphere}}}{4\pi\varepsilon r^2}\right) = \left(\frac{0.001\,\text{C}}{8.85\times10^{-12}\,\frac{\text{F}}{\text{m}}}\right)\left(-\frac{0.001\,\frac{\text{C}}{\text{m}^2}}{2} + \frac{0.001\,\text{C}}{4\pi(5\,\text{m})^2}\right)$$

$$= -5.61\times10^{4}\,\text{N}$$

Electrostatics

Electric Flux – Gauss' Law

$$Q_{\text{encl}} = \oiint_S \epsilon\,\mathbf{E}\cdot d\mathbf{S} \qquad 43.24$$

If **E** is constant and parallel to *d* **S**, then

$$Q_{\text{encl}} = \int_S \varepsilon \mathbf{E} \cdot d\mathbf{S} = \varepsilon E \int d\mathbf{S}$$

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Electrostatics

Work (W) done by moving a charge Q_1 radially from distance r_1 to r_2 in an electric field:

$$W = -Q_1 \int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{L} \qquad 43.25$$

- For a uniform field, the work done by moving a charge Q a distance d parallel to the uniform field:

$$\begin{aligned} W &= -\mathbf{F} \cdot \mathbf{d} = -EQd \\ &= \frac{-V_{\text{plates}} Qd}{r} \\ &= -Q\Delta V \qquad 43.27 \end{aligned}$$

Note that Eq. 43.27 is always true, for all fields. (V may not be easy to compute.)

Electrostatics

Voltage

- A scalar quantity that describes the electrical field.
- The field **E** is the gradient of the voltage, *V*.
- The voltage differential between two points is the work to bring a unit charge from one point to the other.
- The choice of zero potential is arbitrary.
- Electric field strength between two parallel plates:

$$E = \frac{V}{d} \qquad 43.28$$

Electrostatics

Example (FEIM):

A source at zero potential emits electrons at negligible velocity. An open grid at 18 V is located 0.003 m from the source. At what velocity will the electrons pass through the grid?

(A) 490 m/s
(B) 16 000 m/s
(C) 8.3×10^5 m/s
(D) 2.5×10^6 m/s

Electrostatics

The mass of an electron is 9.11×10^{-31} kg.

The work done by the grid on an electron is equal to the change in kinetic energy of the electron and is equal to the charge on the electron times the change in voltage potential.

$$W = \tfrac{1}{2} m\mathrm{v}^2 = q\Delta V$$

$$\mathrm{v} = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{(2)(1.6022\times 10^{-19}\ \mathrm{C})(18\ \mathrm{V})}{9.11\times 10^{-31}\ \mathrm{kg}}}$$

$$= 2.516\times 10^{6}\ \mathrm{m/s}\ \ \left(2.5\times 10^{6}\ \mathrm{m/s}\right)$$

Therefore, the answer is (D).

Current

Change in charge per unit time

$$i(t) = \frac{dq(t)}{dt} \qquad 43.29$$

Current Density (ρ)

- The density of charge moving per unit time through a volume

Volume Current Density (**J**)

- The vector current density

Magnetism

Magnetic field around a current-carrying wire

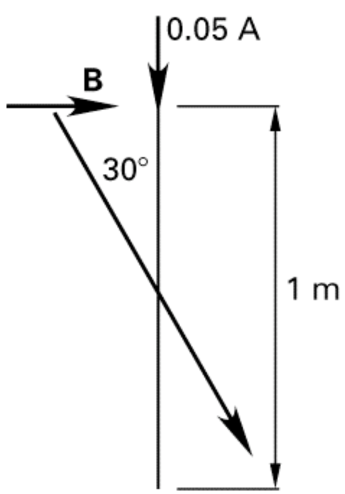
$$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{I}{2\pi r}\,\mathbf{a} \quad \text{[straight wire]} \qquad 43.35$$

Force on current-carrying conductor

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B} \qquad 43.36$$

Magnetism

Example (FEIM):
A magnetic field of 0.0005 T makes a 30° angle with a 1 m wire carrying 0.05 A. What is the force on the wire?
(A) 1.25×10^{-5} N
(B) 5.00×10^{-4} N
(C) 1.25×10^{-3} N
(D) 2.50×10^{-3} N



$$|\mathbf{F}| = I|\mathbf{L} \times \mathbf{B}| = I|\mathbf{L}||\mathbf{B}| \sin\theta$$

$$= (0.05\ \text{A})(1\ \text{m})(0.0005\ \text{T})(\sin 30°)$$

$$= 1.25 \times 10^{-5}\ \text{N}$$

Therefore, the answer is (A).

Induced Voltage

Also called *electromotive force* (emf)

For *N* loops:

$$v = \frac{-N d\phi}{dt} = -NBL\frac{ds}{dt} \qquad 43.41$$

DC Circuits

Resistivity

$$R = \frac{\rho L}{A} \qquad 44.1$$

Depends on temperature:

$$R = R_0(1 + \alpha(T - T_0)) \qquad 44.3$$

$$\rho = \rho_0(1 + \alpha(T - T_0)) \qquad 44.2$$

Example (FEIM):
A cube with an edge length of 0.01 m has resistivity of 0.01 Ω·m.
What is the resistance from one side to the opposite side?

(A) 0.0001
(B) 0.001
(C) 0.1
(D) 1

$$R = \frac{\rho L}{A} = \frac{(0.01\Omega \cdot \mathrm{m})(0.01\mathrm{m})}{(0.01\mathrm{m})^2} = 1\Omega$$

Therefore, the answer is (D).

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DC Circuits

Ohm's Law

$$V = IR \qquad 44.22$$

Resistors in Series: $$R_{eq} = R_1 + R_2 + \cdots + R_n \qquad 44.4$$

Resistors in Parallel: $$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \qquad 44.5$$

Equivalent resistance of two resistors in parallel:

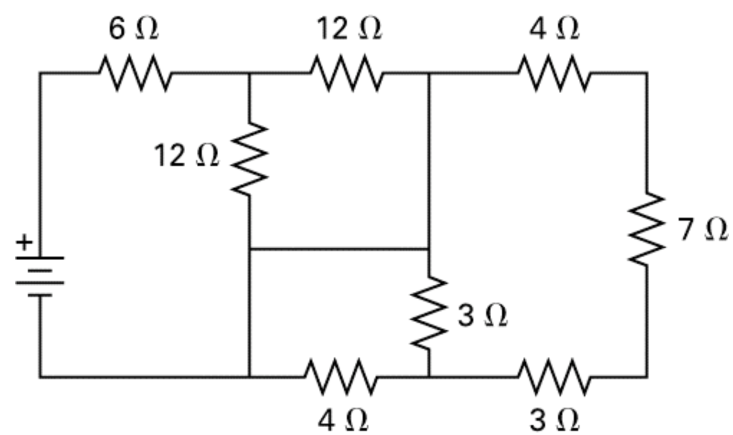
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad 44.6$$

Resistive Power $$P = VI = \frac{V^2}{R} = I^2 R \qquad 44.7$$

DC Circuits

Example (FEIM):

What is the resistance of the following circuit as seen from the battery?



No current will flow through the two 4 Ω resistors, the two 3 Ω resistors, or the 7 Ω resistor. The circuit reduces to one 6 Ω in series with two 12 Ω in parallel.

$$R = 6\ \Omega + 6\ \Omega = 12\ \Omega$$

DC Circuits

Kirchhoff's Laws

- Voltage Law (KVL)

$$\sum V_{\text{rises}} = \sum V_{\text{drops}} \qquad 44.24$$

- Current Law (KCL)

$$\sum I_{\text{in}} = \sum I_{\text{out}} \qquad 44.23$$

DC Circuits

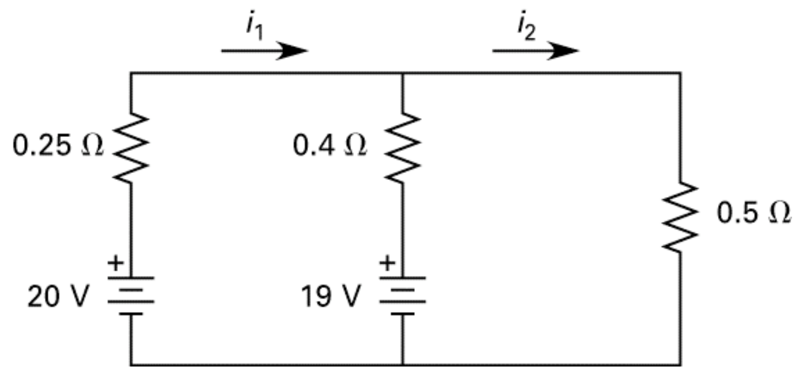
Loop Current Circuit Analysis

1. Select one less than the total number of loops.
2. Write Kirchhoff's voltage equation for each loop.
3. Use the simultaneous equations to solve for the current you want.

Direct Current Electricity

DC Circuits

Example (FEIM):
Find the current through the 0.5 Ω resistor.



The voltage sources around the left loop are equal to the voltage drops across the resistances.

$$20\ \text{V} - 19\ \text{V} = 0.25\ \Omega\ i_1 + 0.4\ \Omega\ (i_1 - i_2)$$

The same is true for the right loop.

$$19\ \text{V} = 0.4\ \Omega\ (i_2 - i_1) + 0.5\ \Omega\ i_2$$

Solve—two equations and two unknowns.

$$0.65\ \Omega\ i_1 - 0.4\ \Omega\ i_2 = 1\ \text{V}$$

$$-0.4\ \Omega\ i_1 + 0.9\ \Omega\ i_2 = 19\ \text{V}$$

$$i_1 = 20\ \text{A}$$

$$i_2 = 30\ \text{A}$$

The current through the 0.5 Ω resistor is 30 A.

DC Circuits

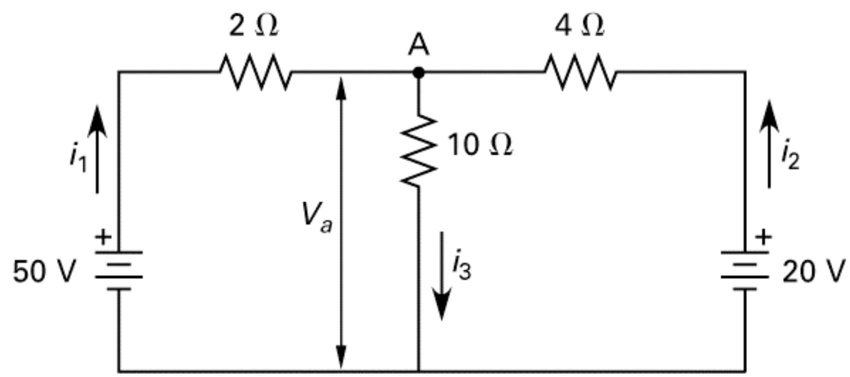
Node Voltage Circuit Analysis

1. Convert all current sources to voltage sources.
2. Choose one node as reference (usually ground).
3. Identify unknown voltages at other nodes compared to reference.
4. Write Kirchhoff's current equation for all unknown nodes except reference node.
5. Write all currents in terms of voltage drops.
6. Write all voltage drops in terms of the node voltages.

DC Circuits

Example (FEIM):

Find the voltage potential at point A and the current i_1.



$$i_1 + i_2 = i_3$$

$$\frac{50\ \text{V} - V_A}{2\ \Omega} + \frac{20\ \text{V} - V_A}{4\ \Omega} = \frac{V_A - 0}{10\ \Omega}$$

$$V_A = 35.3\ \text{V}$$

$$i_1 = \frac{50\ \text{V} - V_A}{2\ \Omega} = \frac{50\ \text{V} - 35.3\ \text{V}}{2\ \Omega}$$

$$= 7.35\ \text{A}$$

Voltage Divider

The voltage across a resistor R in a loop with total resistance R_{total} with a voltage source V is

$$V_R = \frac{R}{R_{\text{total}}} V$$

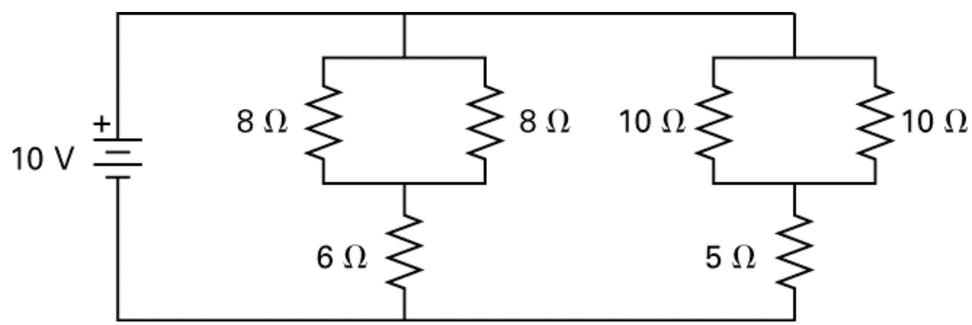
In the general case, the voltage on impedance Z_i in a loop with total impedance Z_{total} with a voltage source v is

$$v_i = \frac{Z_i}{Z_{\text{total}}} \nu$$

NOTE: Each symbol is a complex number in the general case.

Voltage Divider

Example (FEIM):
What is the voltage across the 6 Ω resistor?



(A) 5 V
(B) 6 V
(C) 8 V
(D) 10 V

Two 8 Ω resistors in parallel equal 4 Ω.
The voltage across the 6 Ω resistor is

$$(10\ \text{V})\left(\frac{6\ \Omega}{6\ \Omega + 4\ \Omega}\right) = 6\ \text{V}$$

Therefore, the answer is (B).

Current Divider

The current through a resistor R in parallel with another resistance R_{parallel} and a current into the node of I is:

$$I_R = \frac{R_{\text{parallel}}}{R_{\text{total}}} I$$

(Resistance R does not appear explicitly. R_{total} is the sum of the resistances in parallel.)

In the general case, the current through impedance Z_i connected to a node in parallel with total impedance Z_{total} with a current i into the node is:

$$i_{Z_i} = \frac{Z_{\text{parallel}}}{Z_{\text{total}}} i$$

(Z_{total} is the sum of the impedances in parallel.)

NOTE: Each symbol is a complex number in the general case.

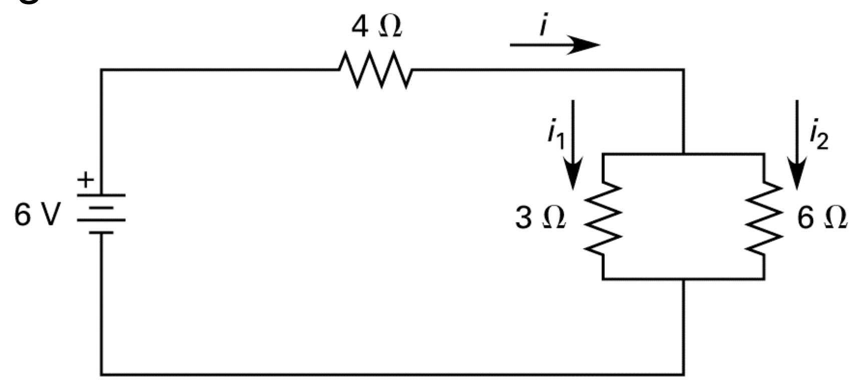
Procedure:

1. Identify the component you want the current through.
2. Simplify the circuit.
3. Determine the current into the node that is connected to the component of interest.
4. Allocate current in proportion to the reciprocal of resistance.

Current Divider

Example (FEIM):

What is the current through the 6 Ω resistor?



(A) 1/10 A
(B) 1/3 A
(C) 1/2 A
(D) 1 A

$i = i_1 + i_2$

Simplify the circuit.

3 Ω in parallel with 6 Ω = 2 Ω
2 Ω in series with 4 Ω = 6 Ω

$$i = \frac{6\ \text{V}}{6\ \Omega} = 1\ \text{A}$$

$R_{\text{parallel}} = 3\ \Omega$
$R_{\text{total}} = 3\ \Omega + 6\ \Omega = 9\ \Omega$

$$i = (1\ \text{A})\left(\frac{3\ \Omega}{3\ \Omega + 6\ \Omega}\right) = 1/3\ \text{A}$$

Therefore, the answer is (B).

Direct Current Electricity

Superposition Theorem

The net current/voltage is the sum of the current/voltage caused by each current/voltage source.

Procedure:

1. Short all voltage sources, and open all current sources, then turn on only one source at a time.
2. Simplify the circuit to get the current/voltage of interest.
3. Repeat until all sources have been used.
4. Add the results for the answer.

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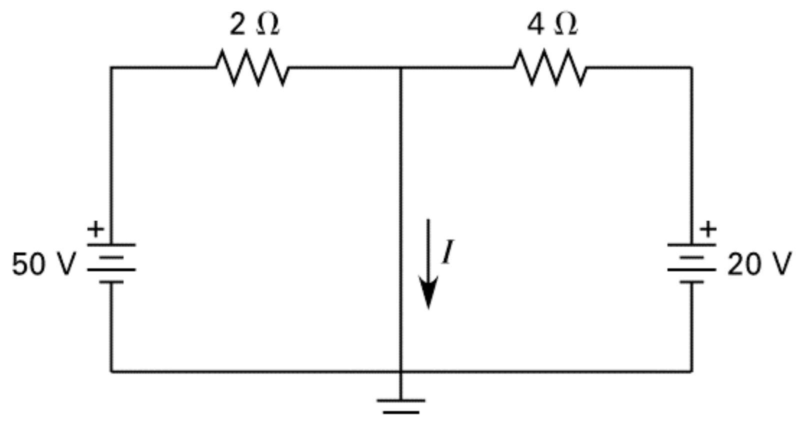
Superposition Theorem

Example (FEIM):

Determine the current through the center leg of the circuit.



Short the 20 V source. $I = 50\frac{V}{2}\Omega = 25\text{ A}$

Short the 50 V source. $I = 20\frac{V}{4}\Omega = 5\text{ A}$

$I_{\text{total}} = 25\text{ A} + 5\text{ A} = 30\text{ A}$

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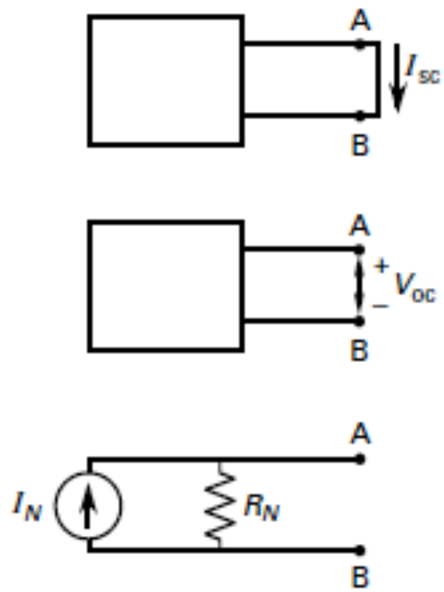
Norton Equivalent

Figure 44.4 Norton Equivalent Circuit



step 1: Measure short-circuit current between terminals A and B.

step 2: Measure open-circuit voltage.

step 3: Draw the Norton equivalent.

$R_N = R_{eq}$

$I_N = I_{sc}$

$$V_{oc} = V_A - V_B \qquad 44.36$$

$$R_{eq} = \frac{V_{oc}}{I_{sc}} \qquad 44.37$$

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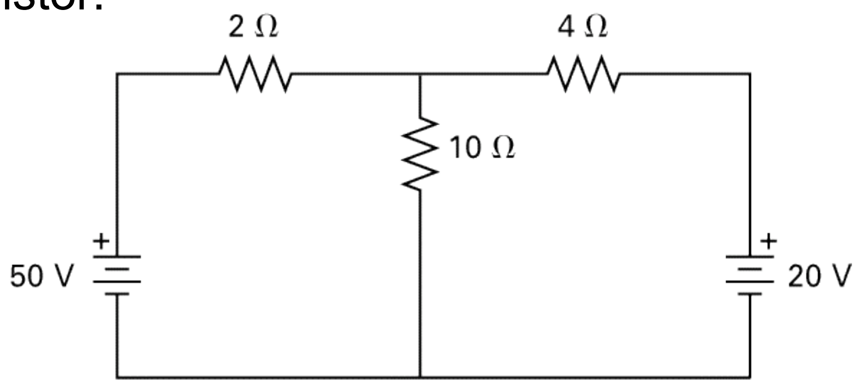
Norton Equivalent

Example (FEIM):

Find the Norton equivalent current and resistance of the circuit as seen by the 10 Ω resistor.



With the 10 Ω resistor open circuited, and the voltage sources shorted, the circuit is 4.0 Ω and 2.0 Ω in parallel.

$$R_N = (2\ \Omega)\left(\frac{4\ \Omega}{2\ \Omega + 4\ \Omega}\right) = 1.33\ \Omega$$

With the 10 Ω resistor shorted, the circuit looks just like the previous example.

$$I_N = 30\text{ A}$$

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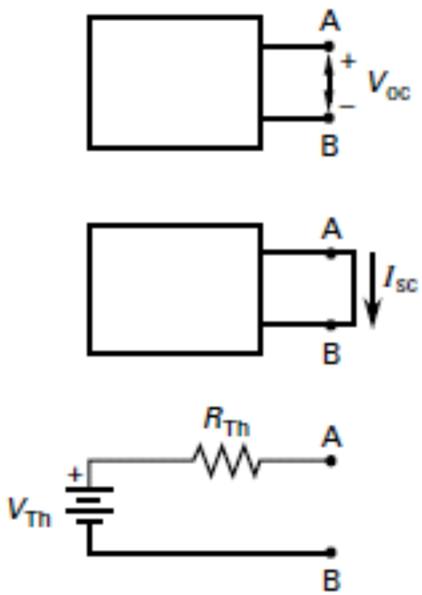
Thevenin Equivalent

Figure 44.3 Thevenin Equivalent Circuit



step 1: Measure open-circuit voltage.

step 2: Measure short-circuit current between terminals A and B.

step 3: Draw the Thevenin equivalent.
$R_{Th} = R_{eq}$
$V_{Th} = V_{oc}$

$$V_{oc} = V_A - V_B \qquad 44.36$$

$$R_N = R_{Th} \qquad 44.38$$

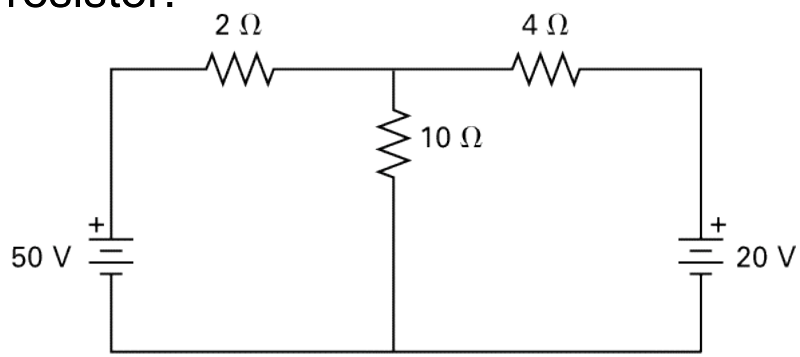
$$V_{Th} = I_N R_N \qquad 44.39$$

$$I_N = \frac{V_{Th}}{R_{Th}} \qquad 44.40$$

Thevenin Equivalent

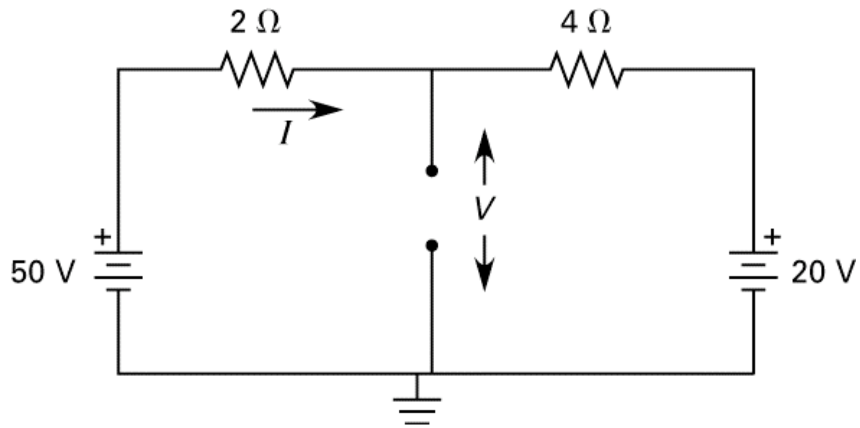
Example (FEIM):

Find the Thevenin equivalent voltage and resistance of the circuit as seen by the 10 Ω resistor.



The Thevenin resistance is the same as the Norton resistance in the previous example, which is 1.3 Ω. With the 10 Ω resistor open-circuited, apply the Kirchhoff voltage law around the loop and find V_{TH} = 40 V.

Thevenin Equivalent



$$(50\ \text{V} - 20\ \text{V}) = I(2\ \Omega + 4\ \Omega)$$

$$I = 5\ \text{A}$$

$$\begin{aligned} V &= 50\ \text{V} - I(2\ \Omega) \\ &= 50\ \text{V} - (5\ \text{A})(2\ \Omega) \\ &= 40\ \text{V} \end{aligned}$$

Capacitors

$$q_C(t) = Cv_C(t) \quad \text{[varying } v(t)\text{]} \qquad 44.9$$

$$\text{energy} = \frac{CV^2}{2} = \frac{VQ}{2}$$

$$= \frac{Q^2}{2C} \qquad 44.13$$

Parallel Plate Capacitors

$$C = \frac{\epsilon A}{d} \qquad 44.10$$

- Capacitance in Parallel:

$$C_{\text{eq}} = C_1 + C_2 + \cdots + C_n \qquad 44.19$$

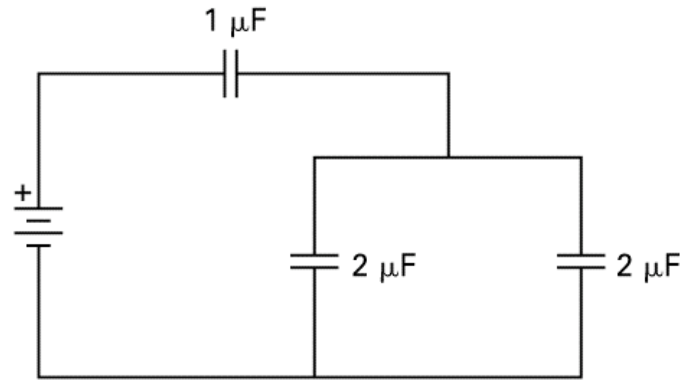
- Capacitance in Series:

$$C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}} \qquad 44.18$$

Capacitors

Example 1 (FEIM):

What is the capacitance seen by the battery?



The two 2 μF capacitors in parallel are equivalent to 4 μF. The 1 μF capacitor in series with the equivalent 4 μF capacitance will add as resistors in parallel.

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{(1\,\mu\text{F})(1\,\mu\text{F})}{1\,\mu\text{F} + 4\,\mu\text{F}}$$

$$= 4/5\,\mu\text{F}$$

Capacitors

Example 2 (FEIM):

A 10 μF capacitor has been connected to a potential source of 150 V. The energy stored in the capacitor in 10 time constants is most nearly

(A) 1.0×10^{-7} J
(B) 9.0×10^{-3} J
(C) 1.1×10^{-1} J
(D) 9.0×10^{1} J

$$\text{Energy} = \frac{(10\times 10^{-6}\ \text{F})(150\ \text{V})^2}{2}$$

$$= 0.11\ \text{J} \quad (1.1\times 10^{-1}\ \text{J})$$

Therefore, the answer is (C).

Inductors

$$v_L(t) = L\frac{dI(t)}{dt} \qquad 44.15$$

$$L = \frac{N\phi}{I} \qquad 44.14$$

$$i_L = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)d\tau \qquad 44.16$$

- Inductance in Series:

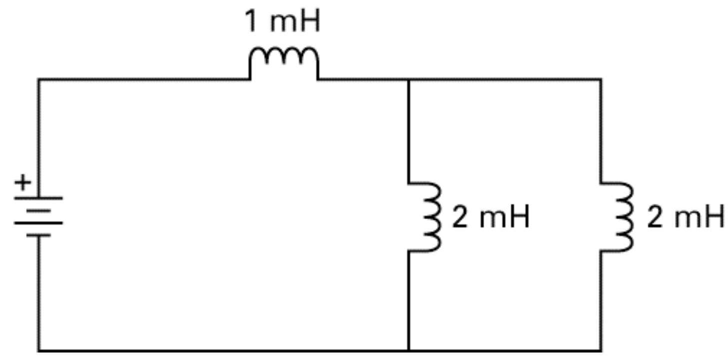
$$L_{\text{eq}} = L_1 + L_2 + \cdots + L_n \qquad 44.20$$

- Inductance in Parallel:

$$L_{\text{eq}} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}} \qquad 44.21$$

Inductors

Example (FEIM):
Find the inductance as seen from the battery.



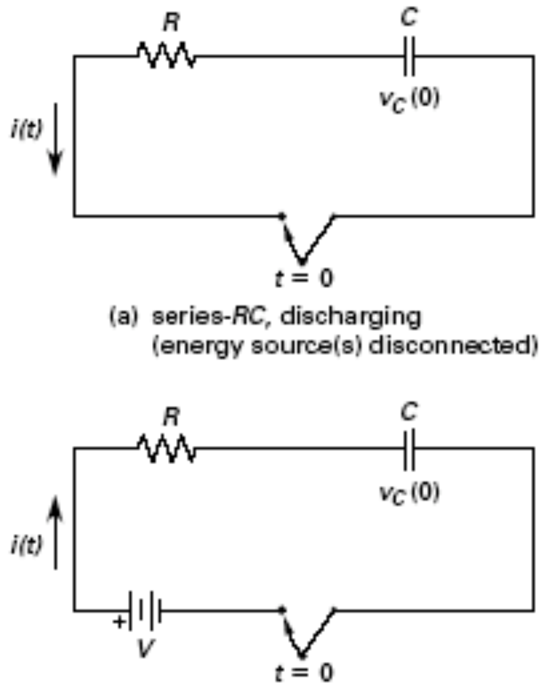
The two 2 mH inductors in parallel will add as resistors in parallel.

$$L = \frac{L_1 L_2}{L_1 + L_2} = \frac{(2\text{ mH})(2\text{ mH})}{2\text{ mH} + 2\text{ mH}} = 1\text{ mH}$$

The 1 mH inductor in series with the equivalent 1 mH inductance will combine for 2 mH total inductance.

RC Transients

Figure 44.5 RC Transient Circuit



(a) series-RC, discharging (energy source(s) disconnected)

(b) series-RC, charging (energy source(s) connected)

$$v_C(t) = v_C(0)e^{-t/RC} + V\left(1 - e^{-t/RC}\right) \quad 44.45$$

$$i(t) = \left(\frac{V - v_C(0)}{R}\right)e^{-t/RC} \quad 44.46$$

$$\begin{aligned} v_R(t) &= i(t)R \\ &= (V - v_C(0))e^{-t/RC} \end{aligned} \quad 44.47$$

RC Transients

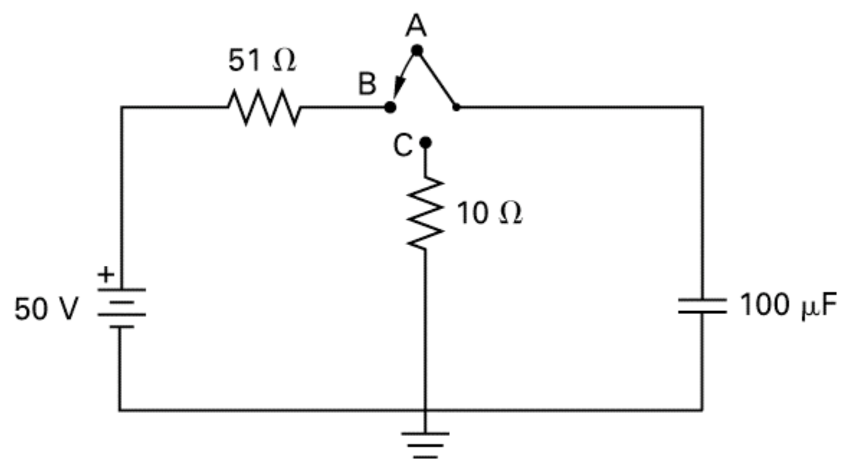
Example (FEIM):

At t = 0, the capacitor is discharged, and the switch is moved from A to B. At t = 6 s, the switch is moved to C.

(a) What is the capacitor voltage at t = 6 s?

(b) What is the current at t = 10 s?

(c) When (after 6 s) is the voltage across the capacitor equal to 10 V?



RC Transients

(a) From time = 0 to 6 s,

V_c (0) = 0

V = 50 V

RC = (51 x 10^3 Ω)(100 x 10^{-6} F) = 5.1 s

$$\nu_c(6\text{ s}) = 0e^{-\frac{6\text{ s}}{5.1\text{ s}}} + 50\text{ V}\left(1 - e^{-\frac{6\text{ s}}{5.1\text{ s}}}\right) = 34.58\text{ V}$$

So 34.58 V is the peak voltage the capacitor reaches before it starts to discharge.

RC Transients

(b) From time = 6 s on,

V_c (6 s) = 34.58 V

V = 0 V

RC = (10 x 10^3 Ω)(100 x 10^{-6} F) = 1 s

$$i(10\text{ s}) = \left(\frac{0 - 34.58\text{ V}}{10 \times 10^3\ \Omega}\right) e^{-\frac{10\text{ s} - 6\text{ s}}{1}} = -6.3 \times 10^{-5}\text{ A}$$

(c) $$v_c(t) = 34.58\text{ V}e^{-\frac{t-6\text{ s}}{1\text{ s}}} + 0\left(1 - e^{-\frac{t-6\text{ s}}{1\text{ s}}}\right) = 10\text{ V}$$

Take the natural logarithm of both sides of the equation

$$\ln 34.58\text{ V } e^{-(t-6\text{ s})} = \ln 10$$

$$\ln e^{-(t-6\text{ s})} + \ln 34.58\text{ V} = \ln 10$$

$$-(t - 6\text{ s}) = \ln 10 - \ln 34.58\text{ V}$$

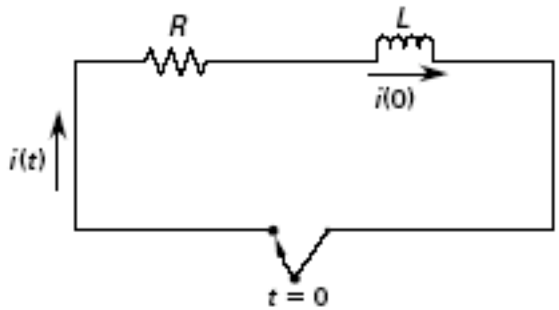
$$t - 6\text{ s} = 1.24\text{ s}$$

$$t = 7.24\text{ s}$$

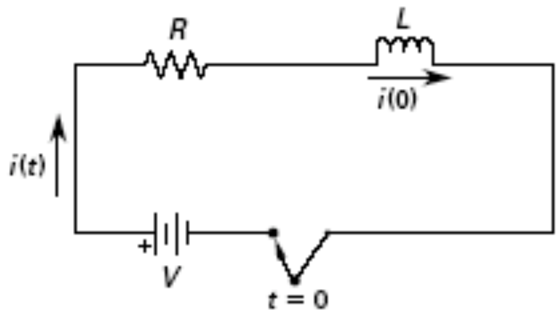
Direct Current Electricity

RL Transients

Figure 44.6 RL Transient Circuit



(a) series-RL, discharging
(energy source(s) disconnected)



(b) series-RL, charging
(energy source(s) connected)

$$v_R(t) = i(t)R$$

$$= i(0)Re^{-Rt/L} + V\left(1 - e^{-Rt/L}\right) \quad 44.41$$

$$i(t) = i(0)e^{-Rt/L} + \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad 44.42$$

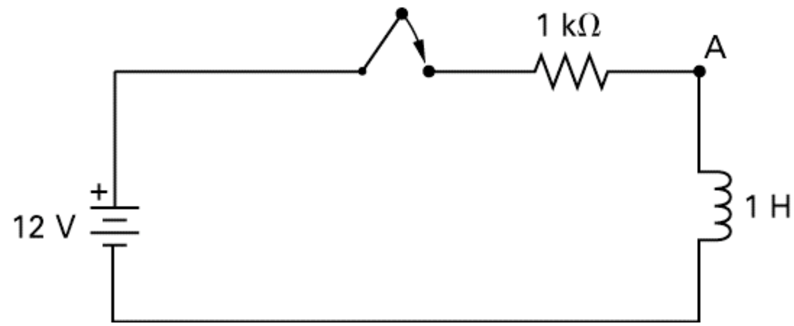
$$v_L(t) = L\frac{di}{dt}$$

$$= -i(0)Re^{-Rt/L} + Ve^{-Rt/L} \quad 44.43$$

RL Transients

Example (FEIM):

Find the voltage at point A at the instant the switch is closed. The switch has been open for a long time, and there is no initial current in the inductor.



(A) 0 V
(B) 1 V
(C) 3 V
(D) 12 V

$i(0) = 0$

$V = 12$ V, so at $t = 0$

$v_L(0^+) = 0Re^0 + (12\text{ V})e^0 = 12\text{ V}$

Therefore, the answer is (D).

Transducers

A transducer is any device used to convert a physical phenomenon into an electrical signal (e.g., microphone, thermocouple, and voltmeter).

Characteristics of measurement design:

- Sensitivity
- Linearity
- Accuracy
- Precision
- Stability

Resistance Temperature Detectors (RTDs)

Make use of changes in their resistance to determine the changes in temperature.

$$R_T \approx R_0\left(1+\alpha\left(T-T_0\right)\right) \qquad 49.3$$

Example (FEIM):

A resistance temperature detector (RTD) that is not perfectly linear is used for a temperature measurement. The temperature coefficient is 3.900×10^{-3}ºC^{-1}, the reference temperature is 0ºC, and the reference resistance is 500.0 Ω. The resistance measured when the actual temperature is 400ºC is 1247 Ω. Determine the error in the temperature measurement.

Resistance Temperature Detectors (RTDs)

If the RTD was perfectly linear the resistance would be given by

$$R_T = R_0\left(1+\alpha(T-T_0)\right)$$

However, the temperature that the RTD indicates is not the actual temperature of 400ºC, so rearranging the RTD equation to solve for the temperature that the RTD indicates yields

$$T = \frac{R_T - R_0 + \alpha R_0 T_0}{\alpha R_0}$$

$$= \frac{1247\ \Omega - 500.0\ \Omega + (3.900\times 10^{-3}\ {}^\circ\mathrm{C}^{-1})(500^\circ\mathrm{C})(0^\circ\mathrm{C})}{(3.900\times 10^{-3}\ {}^\circ\mathrm{C}^{-1})(500^\circ\Omega)}$$

$$= 383.1^\circ\mathrm{C}$$

The error in the measurement is:
Error = 383.1ºC – 400ºC = –16.9ºC

Strain Gages

Metal or semiconductor foils that change resistance linearly with the strain.

$$\mathrm{GF} = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = \frac{\frac{\Delta R}{R}}{\epsilon} \qquad 49.6$$

Example (FEIM):
A strain gage is measured to determine the gage factor. A strain gage with an initial resistance of 200.00 Ω and final resistance of 199.79 Ω when subjected to a strain that causes the gage to compress to 0.9994 cm. The initial length of the gage was 1.0000 cm. What is the gage factor?
(A) 0.15
(B) 0.42
(C) 1.8
(D) 4.0

Strain Gages

$$\text{GF} = \frac{\dfrac{\Delta R}{R}}{\dfrac{\Delta L}{L}}$$

$$= \frac{\dfrac{199.79\ \Omega - 200.00\ \Omega}{200.00\ \Omega}}{\dfrac{0.9994\ \text{cm} - 1.0000\ \text{cm}}{1.0000\ \text{cm}}}$$

$$= 1.75$$

Therefore, the answer is (C).

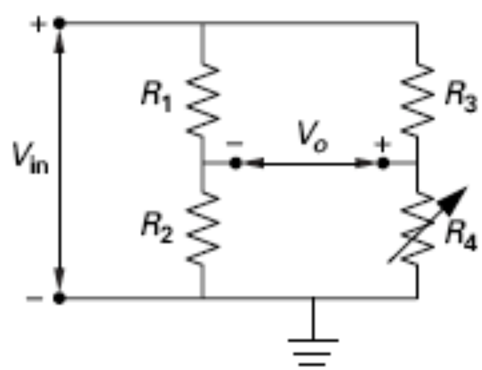
Wheatstone Bridges

Balanced:

$$\frac{R_4}{R_1} = \frac{R_3}{R_2} \quad \text{[balanced]} \qquad 49.11$$

Quarter-bridge:

Figure 49.3 Wheatstone Quarter Bridge



$$R_1 = R_2 = R_3 = R \qquad 49.12$$

$$R_4 = R + \Delta R \qquad 49.13$$

$$|\Delta R| << R \qquad 49.14$$

$$V_o \approx \left(\frac{\Delta R}{4R}\right) V_{\text{in}} \qquad 49.15$$

Wheatstone Bridges

Example (FEIM):
There are three high-precision resistors known to be 10.00 kΩ in a quarter bridge circuit. R_1 is a sensor with a small resistance difference from 10 kΩ. Find the resistance if V_{in} = 5.00 V and V_o = 0.03 V.

$$\Delta R = \frac{4RV_o}{V_{in}}$$

$$= \frac{(4)(10.00\ \text{k}\Omega)(0.03\ \text{V})}{5.00\ \text{V}}$$

$$= 0.24\ \text{k}\Omega$$

$$R_1 = 10.00\ \text{k}\Omega + 0.24\ \Omega = 10.24\ \text{k}\Omega$$

For the strain gage quarter-bridge circuit, ΔR can be substituted.

$$V_o = \frac{1}{4}(\text{GF})\varepsilon V_{in}$$

Sampling

Sampling Rate or Frequency: $f_s = \frac{1}{\Delta t}$ *49.16*

Shannon's Sampling Theorem

Determines the sampling rate to reproduce accurately in the discrete time system.

Nyquist Rate: $f_N = 2f_I$ *49.17* (where f_I is the frequency of interest)

Reproducible Sampling:

$f_s > f_n$ [reproducible sampling]

Sampling

Example (FEIM):

An analog signal is to be sampled at 0.03 μs intervals. What is most nearly the highest frequency that can be accurately reproduced?

(A) 4.0×10^6 Hz
(B) 12×10^6 Hz
(C) 16×10^6 Hz
(D) 18×10^6 Hz

The sampling frequency is

$$f_s = \frac{1}{\Delta t} = \frac{1}{0.03 \times 10^{-6}} = 33 \times 10^6 \text{ Hz}$$

Sampling

The sampling frequency must be greater than the Nyquist rate for accurate reproduction.

$$f_s > 2f_N$$

The greatest frequency that can be reproduced at this sampling rate is

$$f_N < \frac{f_s}{2} = \frac{33 \times 10^6 \text{ Hz}}{2} = 16.7 \times 10^6 \text{ Hz} \quad (16 \times 10^6 \text{ Hz})$$

Therefore, the answer is (C).

Analog-to-Digital Conversion

Voltage Resolution

The range from a high voltage, V_H, and a low voltage, V_L, is divided up into the 2^n ranges.

$$\varepsilon_V = \frac{V_H - V_L}{2^n} \qquad 49.19$$

For example, if all the bits are "1" then the analog value is somewhere between V_H and $V_H - \varepsilon_V$. To calculate the analog value from the digital value use

$$V = \epsilon_V N + V_L \qquad 49.20$$

Analog-to-Digital Conversion

Example (FEIM):
A 16 bit analog-to-digital conversion has a resolution of 1.52588 × 10^{-4} V and the lowest voltage measured has half the magnitude of the highest voltage. Both the high and low voltages are positive. Determine the highest voltage.

$$V_H - V_L = 2^n \varepsilon_v$$
$$= (2)^{16}(1.52588 \times 10^{-4} \text{ V})$$
$$= 10.0 \text{ V}$$

The problem statement also says that

$$|V_H| = |2V_L|$$

Since V_H and V_L are positive, this equation becomes

$$V_H = 2V_L$$

Substituting into the first equation

$$2V_L - V_L = 10 \text{ V}$$
$$V_L = 10 \text{ V}$$
$$V_H = 20 \text{ V}$$

Measurement Uncertainty

Kline-McClintock Equation:

A method for estimating the uncertainty in a function that depends on more than one measurement.

$$w_R = \sqrt{\left(w_1 \frac{\partial f}{\partial x_1}\right)^2 + \left(w_2 \frac{\partial f}{\partial x_2}\right)^2 + \cdots + \left(w_n \frac{\partial f}{\partial x_n}\right)^2} \qquad 49.21$$

Measurement Uncertainty

Example (FEIM):

A function is given by $R = 5x_1 - 3x_2^2$.

Find the measurement uncertainty at (1, 2) if the uncertainty in the variables is ±0.01 and ±0.03 respectively.

$$\frac{\partial R}{\partial x_1} = 5 \qquad \frac{\partial R}{\partial x_2} = -6x_2$$

At the point (1, 2) the partial derivatives are

$$\left.\frac{\partial R}{\partial x_1}\right|_{(1,2)} = 5 \qquad \left.\frac{\partial R}{\partial x_2}\right|_{(1,2)} = (-6)(2) = -12$$

$$w_R = \sqrt{\left(w_1 \left.\frac{\partial R}{\partial x_1}\right|_{(1,2)}\right)^2 + \left(w_2 \left.\frac{\partial R}{\partial x_2}\right|_{(1,2)}\right)^2}$$

$$= \sqrt{\left((0.01)(5)\right)^2 + \left((0.03)(-12)\right)^2}$$

$$= 0.36$$

Measurement Uncertainty

If the function R is the sum of the measurements, $R = x_1 + x_2 + x_3 + \ldots + x_n$, then the Kline-McClintock method reduces to

$$w_R = \sqrt{w_1^2 + w_2^2 + \cdots + w_n^2}$$

This is called the *root sum square* (RSS) value.

If the function R is the sum of the measurements times constants, $R = a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n$, then the Kline-McClintock method reduces to

$$w_R = \sqrt{a_1^2w_1^2 + a_2^2w_2^2 + \cdots + a_n^2w_n^2}$$

This is called a *weighted RSS* value.