Dynamics

Overview

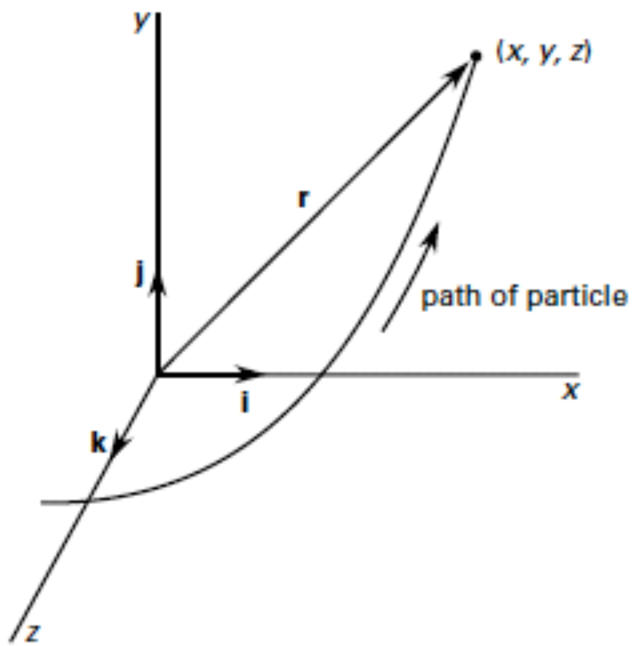
Dynamics—the study of moving objects.

Kinematics—the study of a body's motion independent of the forces on the body.

Kinetics—the study of motion and the forces that cause motion.

Kinematics—Rectangular Coordinates

Figure 14.1 Rectangular Coordinates



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad 14.7$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

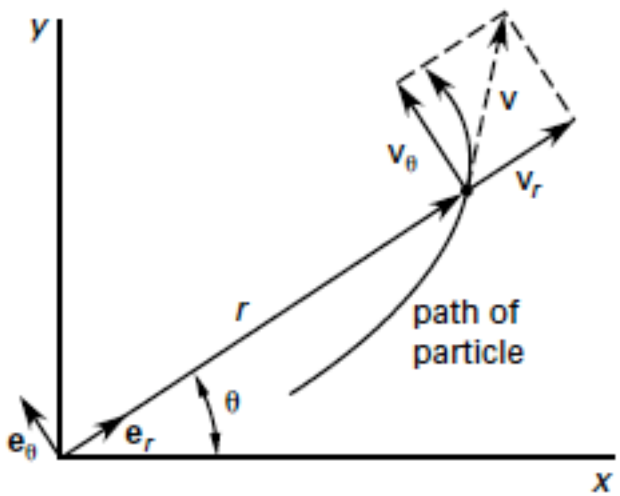
$$= \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k} \qquad 14.8$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$= \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j} + \ddot{z}\,\mathbf{k} \qquad 14.9$$

Kinematics—Polar Coordinates

Figure 14.2 Radial and Transverse Coordinates



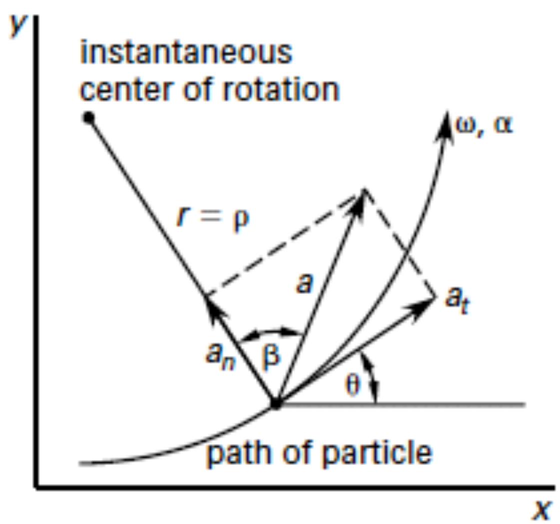
$$\mathbf{r} = r\mathbf{e}_r \quad \text{[position]} \qquad 14.14$$

$$\begin{aligned}\mathbf{v} &= \mathrm{v}_r\mathbf{e}_r + \mathrm{v}_\theta\mathbf{e}_\theta \\ &= \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \quad \text{[velocity]} \qquad 14.15\end{aligned}$$

$$\begin{aligned}\mathbf{a} &= a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta \\ &= (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r \\ &\quad + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \quad \text{[acceleration]} \qquad 14.16\end{aligned}$$

Kinematics—Polar Coordinates

Figure 14.3 Tangential and Normal Coordinates



$$\mathbf{v} = \mathrm{v}_t \mathbf{e}_t \qquad 14.17$$

$$\mathbf{a} = \left(\frac{d\mathrm{v}_t}{dt}\right)\mathbf{e}_t + \left(\frac{\mathrm{v}_t^2}{\rho}\right)\mathbf{e}_n \qquad 14.18$$

Kinematics—Circular Motion

$$\theta \qquad \text{[angular position]} \qquad 14.19$$

$$\omega = \frac{d\theta}{dt} \qquad \text{[angular velocity]} \qquad 14.20$$

$$\alpha = \frac{d\omega}{dt}$$

$$= \frac{d^2\theta}{dt^2} \qquad \text{[angular acceleration]} \qquad 14.21$$

Dynamics

Kinematics—Circular Motion

Angular velocity = $\omega = \dot{\theta} = \frac{v_t}{r}$

Angular acceleration = $\alpha = \dot{\omega} = \ddot{\theta} = \frac{a_t}{r}$

Tangential acceleration = $a_t = r\alpha = \frac{d\text{v}_t}{dt}$ 14.24

Normal acceleration = $a_n = \frac{\text{v}_t^2}{r} = r\omega^2$ 14.25

Kinematics—Circular Motion

Example (FEIM):

A turntable starts from rest and accelerates uniformly at 1.5 rad/s^2. How many revolutions does it take for the rotational frequency to reach 33.33 rpm?

$$\omega = 2\pi f = \left(2\pi\,\frac{\text{rad}}{\text{rev}}\right)\left(33.33\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 3.49\,\frac{\text{rad}}{\text{s}}$$

$$\theta = \int_0^{t_f} \omega\, dt = \int_0^{t_f} \alpha t\, dt = \frac{\alpha t_f^2}{2}$$

$$t = \frac{\omega}{\alpha}$$

$$\theta = \frac{\omega^2}{2\alpha} = \frac{\left(3.49\,\frac{\text{rad}}{\text{s}}\right)^2}{(2)\left(1.5\,\frac{\text{rad}}{\text{s}^2}\right)} = 4.06\ \text{rad}$$

$$n = \frac{4.06\ \text{rad}}{2\pi\,\frac{\text{rad}}{\text{rev}}} = 0.65\ \text{revolution}$$

Kinematics—Projectile Motion

Constant acceleration formulas:

$$s = s_0 + v_0 t + \tfrac{1}{2} a t^2$$

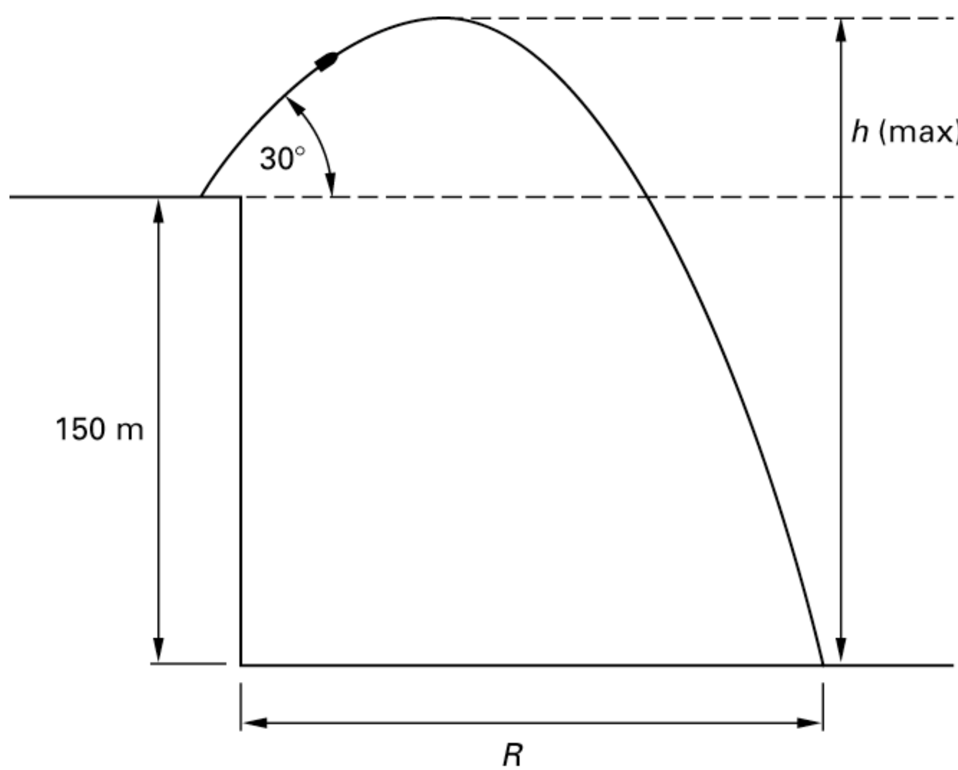
$$v = v_0 + a_0 t$$

$$v^2 = v_0^2 + 2a_0 \left(s - s_0 \right)$$

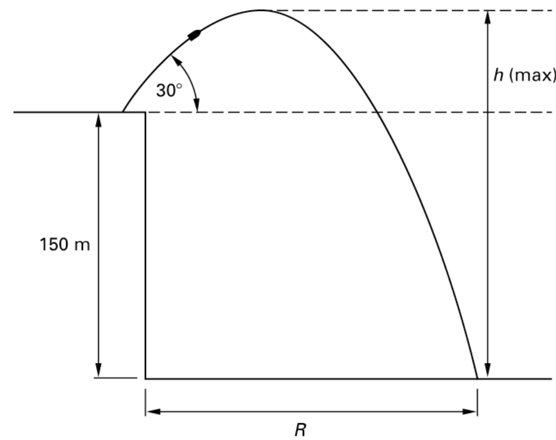
Kinematics—Projectile Motion

Example 1 (FEIM):

A projectile is launched at 180 m/s at a 30° incline. The launch point is 150 m above the impact plane. Find the maximum height, flight time, and range.



Kinematics—Projectile Motion



$$h'_{max} = v_0 \sin\theta + at = 0$$

$$t_{max} = -v_0 \frac{\sin\theta}{a} = -\frac{\left(180\,\frac{m}{s}\right)(0.5)}{\left(-9.8\,\frac{m}{s}\right)} = 9.18\ s$$

$$h = h_0 + v_0 t \sin\theta + \tfrac{1}{2}at^2$$

$$h_{max} = 150\ m + \left(180\,\frac{m}{s^2}\right)(9.18\ s)(0.5) - \left(\tfrac{1}{2}\right)\left(9.8\,\frac{m}{s^2}\right)(9.18\ s)^2 = 563\ m$$

$$h_{impact} = 0 = 150\ m + 90t - 4.9t^2$$

By the quadratic equation, $t_{impact} = 19.9\ s$

$$\cos 30° = 0.866$$

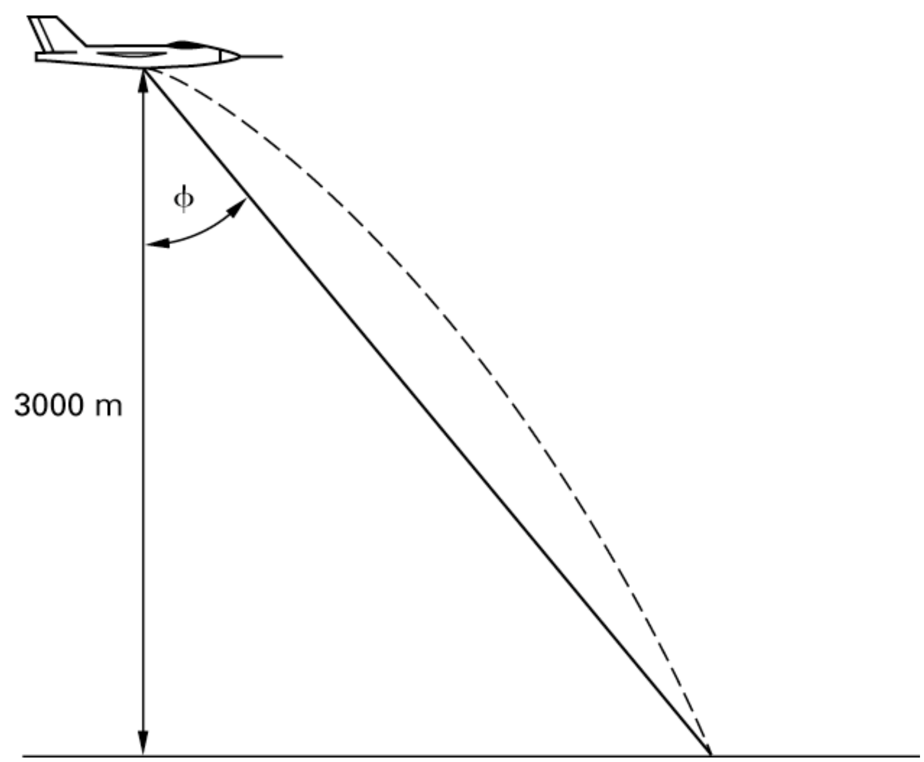
$$R_{impact} = R_0 + v_0 t_{impact} \cos\theta$$

$$= 0 + \left(180\,\frac{m}{s}\right)(19.9\ s)(0.866) = 3100\ m$$

Kinematics—Projectile Motion

Example 2 (FEIM):

A bomber flies horizontally at 275 km/h and an altitude of 3000 m. At what viewing angle from the bomber to the target should the bomb be dropped?



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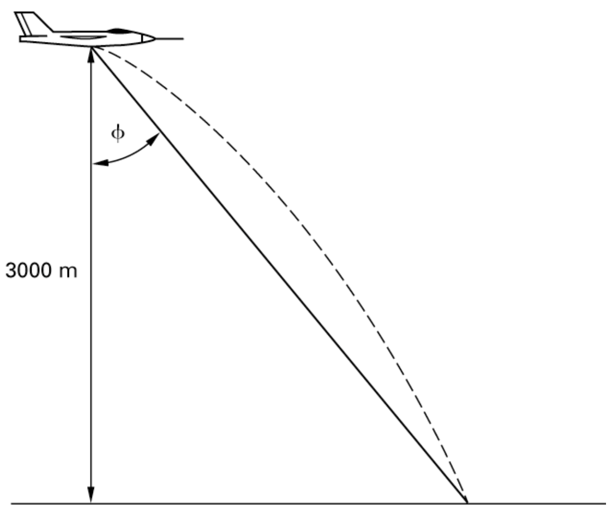
Kinematics—Projectile Motion



$$h_{\text{impact}} = \tfrac{1}{2}at^2 = \left(\tfrac{1}{2}\right)\left(-9.8\,\frac{\text{m}}{\text{s}^2}\right)t^2 = -3000\text{ m}$$

$$t = \sqrt{\frac{2h_{\text{impact}}}{g}} = 24.7\text{ s}$$

$$\left(275\,\frac{\text{km}}{\text{hr}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 76.39\text{ m/s}$$

$$R_{\text{impact}} = R_0 + \text{v}_0 t = 0 + \left(76.39\,\frac{\text{m}}{\text{s}}\right)(24.7\text{ s}) = 1887\text{ m}$$

$$\phi = \arctan\frac{R_{\text{impact}}}{h_{\text{impact}}} = \arctan\frac{1887\text{ m}}{3000\text{ m}} = 32.2^\circ$$

Kinetics—Newton's 2nd Law of Motion

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad 15.2$$

For a constant mass,

$$\begin{aligned} \mathbf{F} &= \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} \\ &= m\frac{d\mathbf{v}}{dt} \\ &= m\mathbf{a} \qquad \text{[SI]} \qquad 15.3a \end{aligned}$$

One-dimension motion

$$s(t) = \int \mathrm{v}(t)dt = \int\int a(t)dt^2 \qquad 14.4$$

$$\mathrm{v}(t) = \frac{ds(t)}{dt} = \int a(t)dt \qquad 14.5$$

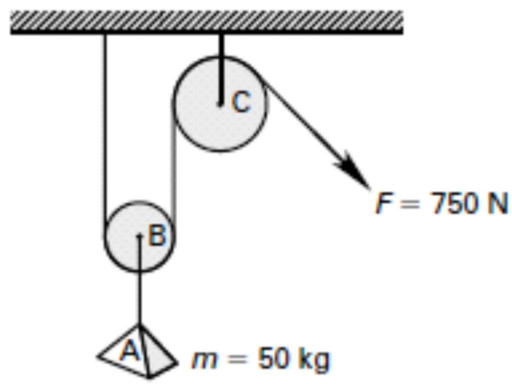
$$a(t) = \frac{d\mathrm{v}(t)}{dt} = \frac{d^2s(t)}{dt^2} \qquad 14.6$$

Kinetics—Newton's 2nd Law of Motion

Example (FERM prob. 6, p. 15-5):

Problem 6

A constant force of 750 N is applied through a pulley system to lift a mass of 50 kg as shown. Neglecting the mass and friction of the pulley system, what is the acceleration of the 50 kg mass?



(A) 5.20 m/s^2
(B) 8.72 m/s^2
(C) 16.2 m/s^2
(D) 20.2 m/s^2

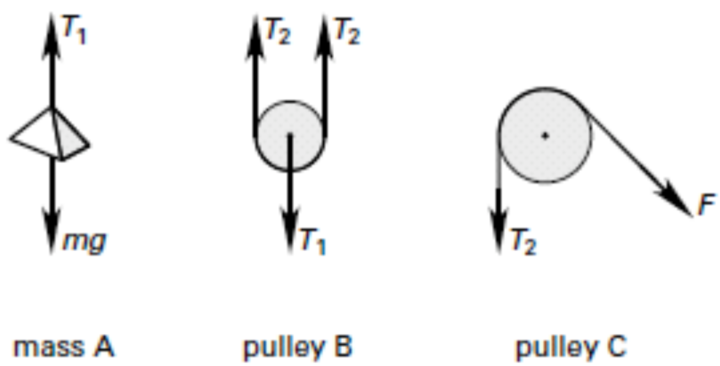
Dynamics

Kinetics—Newton's 2nd Law of Motion

Example (FERM prob. 6, p. 15-5):

Solution

Apply Newton's second law to the mass and to the two frictionless, massless pulleys. Refer to the following free-body diagrams.



mass A: $T_1 - mg = ma$

pulley B: $2T_2 - T_1 = 0$

pulley C: $T_2 = F = 750\ \text{N}$

$$a = \frac{T_1 - mg}{m} = \frac{2T_2 - mg}{m} = \frac{2F - mg}{m}$$

$$= \frac{(2)(750\ \text{N}) - (50\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{50\ \text{kg}}$$

$$= 20.2\ \text{m/s}^2$$

Answer is D.

Kinetics—Impulse and Momentum

Impulse (constant mass in one dimension)

$$\mathbf{Imp} = \int_{t_1}^{t_2} \mathbf{F}\,dt \qquad 17.9$$

Momentum

$$\mathbf{p} = m\mathbf{v} \qquad \text{[SI]} \qquad 15.1a$$

Impulse-Momentum Principle

$$\mathbf{Imp} = \Delta\mathbf{p} \qquad 17.11$$

Kinetics—Impulse and Momentum

Example 1 (FEIM):

A 0.046 kg marble attains a velocity of 76 m/s in a slingshot. Contact with the slingshot is 1/25 of a second. What is the average force on the marble during the launch?

$$F_{\text{ave}} = \frac{m\Delta \text{v}}{\Delta t} = \frac{(0.046\ \text{kg})\left(76\ \frac{\text{m}}{\text{s}}\right)}{0.04\ \text{s}} = 87.4\ \text{N}$$

Kinetics—Impulse and Momentum

Example 2 (FEIM):

A 2000 kg cannon fires a 10 kg projectile horizontally at 600 m/s. It takes 0.007 s for the projectile to pass through the barrel. What is the recoil velocity if the cannon is not restrained? What average force must be exerted on the cannon to keep it from moving?

$$m_{\text{projectile}}\Delta \text{v}_{\text{projectile}} = m_{\text{cannon}}\Delta \text{v}_{\text{cannon}}$$

$$(10\ \text{kg})\left(600\,\frac{\text{m}}{\text{s}}\right) = (2000\ \text{kg})(\text{v}_{\text{cannon}})$$

$$\text{v}_{\text{cannon}} = 3\,\frac{\text{m}}{\text{s}} = \text{initial recoil velocity}$$

$$F = \frac{m\Delta \text{v}}{\Delta t} = \frac{(10\ \text{kg})\left(600\,\frac{\text{m}}{\text{s}}\right)}{0.007\ \text{s}} = 8.57\times 10^{5}\ \text{N}$$

Dynamics

Work & Energy

Work

$$W = \int \mathbf{F} \cdot d\mathbf{r} \qquad 17.1$$

$$W = E_2 - E_1 \qquad 17.8$$

Kinetic Energy of a Mass

$$\mathrm{KE} = \frac{1}{2} m \mathrm{v}^2 \quad \text{[SI]} \qquad 17.2a$$

$$W = \mathrm{KE}_2 - \mathrm{KE}_1 = \tfrac{1}{2} m(\mathrm{v}_2^2 - \mathrm{v}_1^2)$$

Kinetic Energy of a Rotating Body

$$\mathrm{KE} = \frac{1}{2} I \omega^2 \quad \text{[SI]} \qquad 17.3a$$

$$W = \mathrm{KE}_2 - \mathrm{KE}_1 = \tfrac{1}{2} I(\omega_2^2 - \omega_1^2)$$

Potential Energy

- Gravity

$$\mathrm{PE} = mgh \quad \text{[SI]} \qquad 17.5a$$

$$W = \mathrm{PE}_2 - \mathrm{PE}_1 = mg(h_2 - h_1)$$

- Spring (linear)
  $F_s = kx$ where the spring is compressed a distance x

$$\mathrm{PE} = \frac{1}{2} k x^2 \qquad 17.6$$

$$W = \mathrm{PE}_2 - \mathrm{PE}_1 = \tfrac{1}{2} k(x_2^2 - x_1^2)$$

Work & Energy

Conservation of Energy

- For a closed system (no external work), the change in potential energy equals the change in kinetic energy.

$$PE_1 - PE_2 = KE_2 - KE_1$$
$$PE_1 + KE_1 = PE_2 + KE_2$$

- For a system with external work, W equals ΔPE + ΔKE.

$$W_{1\rightarrow 2} = (PE_1 - PE_2) + (KE_2 - KE_1)$$
$$PE_1 + KE_1 + W_{1\rightarrow 2} = PE_2 + KE_2$$

Work & Energy

Impacts: Momentum is always conserved.
Elastic Impacts: Kinetic energy is conserved.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

[always true] *17.15*

Example 1 (FEIM):

Two identical balls collide along their centerlines in an elastic collision. The initial velocity of ball 1 is 0.85 m/s. The initial velocity of ball 2 is –0.53 m/s. What is the relative velocity of each ball after the collision?

(A) –0.53 m/s and 0.85 m/s
(B) –0.72 m/s and 1.2 m/s
(C) –5.1 m/s and 1.2 m/s
(D) 0.98 m/s and 1.8 m/s

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 = m_2 so v_1 + v_2 = v_1' + v_2' = 0.85 - 0.53 = 0.32$$

$$\tfrac{1}{2} m v_1^2 + \tfrac{1}{2} m v_2^2 = \tfrac{1}{2} m {v_1'}^2 + \tfrac{1}{2} m {v_2'}^2$$

$$v_1^2 + v_2^2 = {v_1'}^2 + {v_2'}^2$$

Solving two equations and two unkowns:

$$v_1' = -0.53 \text{ m/s}$$

$$v_2' = 0.85 \text{ m/s}$$

Therefore, (A) is correct.

Work & Energy

Example 2 (FEIM):

Ball A of 200 kg is traveling at 16.7 m/s. It strikes stationary ball B of 200 kg along the centerline. What is the velocity of ball A after the collision? Assume the collision is elastic.

(A) –16.7 m/s
(B) –8.35 m/s
(C) 0
(D) 8.35 m/s

$m_A = m_B so v_A + v_B = v'_A + v'_B = 16.7 \text{ m/s}$

$v_A^2 + v_B^2 = {v'_A}^2 + {v'_B}^2$

There are two possible solutions for these equations.

$v'_A = 0,\ v'_B = 16.7 \text{ m/s}$

or

$v'_A = 16.7 \text{ m/s},\ v'_B = 0$

Since there must be a change in the collision, ball A's velocity must be 0. Therefore, (C) is correct.

Work & Energy

Inelastic Impacts:

Kinetic energy does not have to be conserved if some energy is converted to another form.

$$v_1' - v_2' = -e(v_1 - v_2)$$

where e = coefficient of restitution

$$v_1' = \frac{m_2 v_2(1+e) + (m_1 - e m_2)v_1}{m_1 + m_2}$$

$$v_2' = \frac{m_1 v_1(1+e) + (e m_1 - m_2)v_2}{m_1 + m_2}$$

Example 1 (FEIM):

A ball is dropped from an initial height h_o. If the coefficient of restitution is 0.90, how high will the ball rebound?

(A) $0.45h_o$

(B) $0.81h_o$

(C) $0.85h_o$

(D) $0.9h_o$

$$mgh = \tfrac{1}{2}mv^2$$

$$v = \sqrt{2gh_o}$$

$$v' = -ev = -e\sqrt{2gh_o} = \sqrt{2gh'}$$

$$h' = e^2 h_o = (0.9)^2 h_o = 0.81h_o$$

Since there must be a change in the collision, ball A’s velocity must be 0. Therefore, (B) is correct.

Work & Energy

Example 2 (FEIM):

Two masses collide in a perfectly inelastic collision. What is the velocity of the combined mass after collision?

$m_1 = 4m_2 \quad \mathrm{v}_1 = 10 \text{ m/s} \quad \mathrm{v}_2 = -20 \text{ m/s}$

(A) 0

(B) 4 m/s

(C) –5m/s

(D) 10 m/s

"Perfectly inelastic" means the masses collide and stick together.

$$m_1\mathrm{v}_1 + m_2\mathrm{v}_2 = m_3\mathrm{v}_3$$

$$m_3 = m_1 + m_2 = 5m_2$$

$$4m_2\left(10\,\frac{\text{m}}{\text{s}}\right) + m_2\left(-20\,\frac{\text{m}}{\text{s}}\right) = 5m_2\mathrm{v}_3$$

$$5m_2\mathrm{v}_3 = \left(40\,\frac{\text{m}}{\text{s}}\right)m_2 - \left(20\,\frac{\text{m}}{\text{s}}\right)m_2$$

$$\mathrm{v}_3 = 4 \text{ m/s}$$

Therefore, (B) is correct.

Kinetics

Friction

$$F_f = \mu N \qquad 15.5$$

Example (FEIM):

A snowmobile tows a sled with a weight of 3000 N. It accelerates up a 15° slope at 0.9 m/s². The coefficient of friction between the sled and the snow is 0.1. What is the tension in the tow rope?

$$F_{\text{slope}} = F_{\text{rope}} - (F_{\text{friction}} + F_{\text{gravity}}) = ma_{\text{slope}}$$

$$F_{\text{rope}} = F_{\text{friction}} + F_{\text{gravity}} + ma_{\text{slope}}$$

$$= mg\sin 15^\circ + mg\mu\cos 15^\circ + ma_{\text{slope}}$$

$$= (3000)(0.2588) + (3000\text{ N})(0.1)(0.9659)$$

$$+ \left(\frac{3000\text{ N}}{9.8\,\frac{\text{m}}{\text{s}^2}}\right)\left(0.9\,\frac{\text{m}}{\text{s}^2}\right)$$

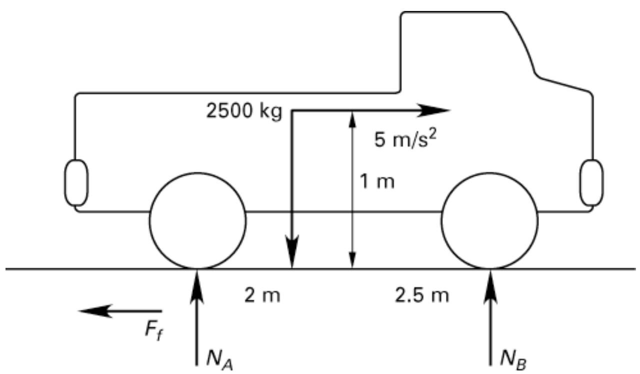
$$= 1342\text{ N}$$

Plane Motion of a Rigid Body

Similar equations can be written for the *y*-direction or any other coordinate direction.

$$F_x = ma_x \qquad \text{[SI]} \qquad 15.8$$

Example (FEIM):
A 2500 kg truck skids with a deceleration of 5 m/s². What is the coefficient of sliding friction? What are the frictional forces and normal reactions (per axle) at the tires?

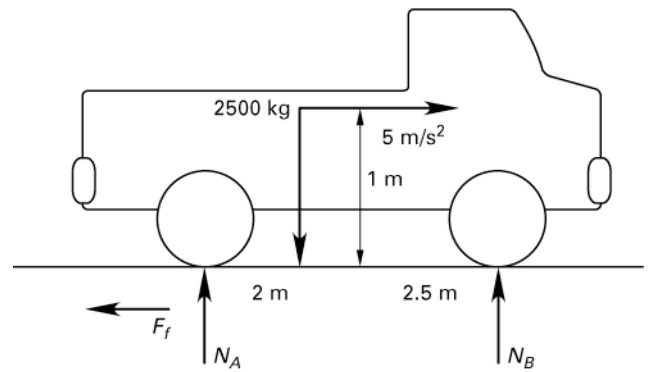


The force of deceleration is equal to the friction force.

$$F_{\text{deceleration}} = (2500\text{ kg})\left(5\frac{\text{m}}{\text{s}^2}\right) = F_{\text{friction}} = \mu mg$$

$$= \mu(2500\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)$$

$$\mu = \left(5\frac{\text{m}}{\text{s}^2}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 0.51$$



The moment about the center of gravity, M_A, must be equal to zero.

$$\sum M_{\text{A}} = (4.5\text{ m})N_B - (2\text{ m})mg - (1\text{ m})(F_{\text{deceleration}}) = 0$$

$$= (4.5\text{ m})N_B - (2\text{ m})(2500\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) - (2500\text{ kg})\left(5\frac{\text{m}}{\text{s}^2}\right) = 0$$

$$N_B = 13{,}667\text{ N}$$

$$N_A = mg - N_B = (2500\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) - 13667\text{ N} = 10833\text{ N}$$

Rotation

Rotation about Fixed Axis

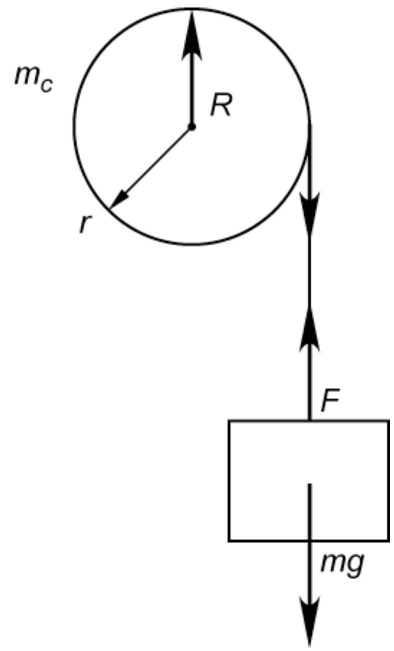
$$\alpha = \frac{M}{I} \quad 16.12$$

$$\omega = \int \alpha dt = \omega_0 + \left(\frac{M}{I}\right) t \quad 16.13$$

$$\theta = \int \int \alpha dt^2 = \theta_0 + \omega_0 t + \left(\frac{M}{2I}\right) t^2 \quad 16.14$$

Rotation

Example (FEIM):
A mass m is attached to a rope wound around a cylinder of mass m_C and radius r. What is the acceleration of the falling mass? What is the rope tension?



There are three simultaneous equations for the movement of the mass, the cylinder, and the relationship between the two:

$$mg - F = ma$$

$$Fr = I\alpha$$

$$\alpha r = a$$

Rotation

Rearranging,

$$Fr = I\frac{a}{r}$$

$$F = \frac{m_c r^2 a}{2r^2} = \frac{m_c}{2}a$$

$$mg - \frac{m_c}{2}a = ma$$

$$a = g\frac{m}{m + \frac{m_c}{2}}$$

Solving for F,

$$F = \frac{m_c}{2}a = g\frac{m_c m}{2\left(m + \frac{m_c}{2}\right)} = \frac{mm_c}{2m + m_c}g$$

Dynamics

Rotation

Centripetal Force

- The force required to keep a body rotating about an axis.

$$F_c = ma_n = \frac{m\mathrm{v}_t^2}{r} = mr\omega^2 \quad \text{[SI]} \qquad \mathit{16.16a}$$

(*r* is the distance from the center of mass to the center of rotation.)

Centrifugal Force

- The “reaction” to centripetal force.
- The centrifugal force, like any inertia force, should not be used in free-body diagrams.

Example (FEIM):

A 2000 kg car travels 65 km/hr around a curve of radius 60 m. What is the centripetal force?

$$65 \text{ km/hr} = 18.06 \text{ m/s}$$

$$F_c = \frac{m\mathrm{v}^2}{r} = \frac{(2000 \text{ kg})\left(18.06 \frac{\text{m}}{\text{s}}\right)^2}{60 \text{ m}} = 10\,900 \text{ N}$$

Rotation

Banking Curves

$$\tan\theta = \frac{v_t^2}{gr} \qquad \textit{16.18}$$

Example (FEIM):
A 2000 kg car travels at 64 km/hr around a banked curve with a radius of 150 m. What should the angle between the roadway and the horizontal be so tire friction is not needed to prevent sliding?

$$\left(64\frac{\text{km}}{\text{hr}}\right)\left(1000\frac{\text{m}}{\text{km}}\right)\left(\frac{1\text{hr}}{3600\text{ s}}\right) = 17.8\frac{\text{m}}{\text{s}}$$

$$\theta = \arctan\left(\frac{v^2}{gr}\right) = \arctan\left(\frac{\left(17.8\frac{\text{m}}{\text{s}}\right)}{\left(9.8\frac{\text{m}}{\text{s}}\right)(150\text{ m})}\right) = 12.1^\circ$$

Rotation

Free Vibration

Spring and Mass: $mg - k(x + \delta_{st}) = m\ddot{x}$ [SI] 15.20

Natural Frequency: $\omega = \sqrt{\dfrac{k}{m}}$ [SI] 15.24a

Solution: $x(t) = C_1 \cos \omega t + C_2 \sin \omega t$ 15.23

For initial conditions $x(0) = x_0$ and $x'(0) = v_0$,

$$x(t) = x_0 \cos \omega t + \left(\frac{v_0}{\omega}\right) \sin \omega t \qquad 15.27$$

For initial conditions $x(0) = x_0$ and $x'(0) = 0$,

$$x(t) = x_0 \cos \omega t \qquad 15.28$$

Torsional Free Vibration:

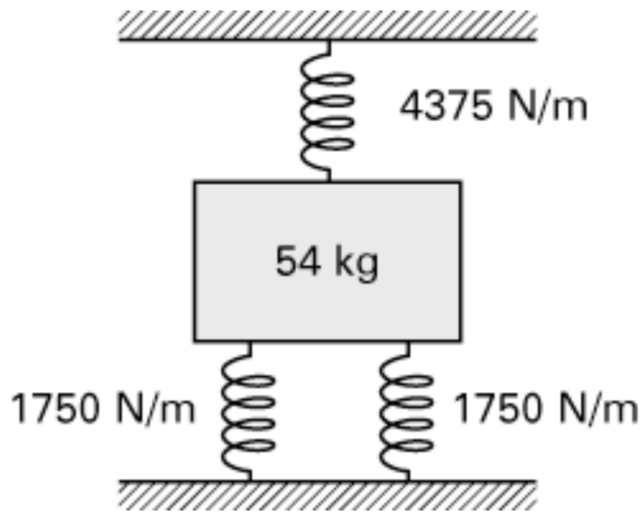
$$\ddot{\theta} + \omega_n^2 \theta = 0 \qquad 16.19$$

Natural frequency: $\omega_n = \sqrt{\dfrac{k_t}{I}}$

Solution: $\theta(t) = \theta_0 \cos \omega_n t + \left(\dfrac{\omega_0}{\omega_n}\right) \sin \omega_n t$ 16.21

Rotation

Example (FEIM):
A 54 kg mass is supported by three springs, as shown. The starting position is 5.0 cm down from the equilibrium position. No external forces act on the mass after it is released. What are the maximum velocity and acceleration?



Rotation

From the solution to the differential equation,

$$x(t) = x_0 \cos \omega_n t + \left(\frac{\mathrm{v}_0}{\omega_n}\right) \sin \omega_n t$$

$$\mathrm{v}(t) = x'(t) = -x_0 \omega_n \sin \omega_n t + \mathrm{v}_0 \cos \omega_n t$$

But, $\mathrm{v}_0 = 0$; so $\mathrm{v}(t) = -x_0 \omega_n \sin \omega_n t$

$$x_0 = 0.05 \text{ m}$$

$$k = k_1 + k_2 + k_3 = 1750 \frac{\text{N}}{\text{m}} + 1750 \frac{\text{N}}{\text{m}} + 4375 \frac{\text{N}}{\text{m}} = 7875 \frac{\text{N}}{\text{m}}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{7875 \ \frac{\text{N}}{\text{m}}}{54 \text{ kg}}} = 12.08 \text{ rad/s}$$

$$v = x' = (-0.05 \text{ m})\left(12.08 \ \frac{\text{rad}}{\text{s}}\right) \sin 12.08t$$

$$= -0.604 \text{ m/s} \ \sin 12.08t$$

The maximum velocity is when sin 12.08t = 1. Because the motion is oscillatory, the maximum velocity occurs in both directions.

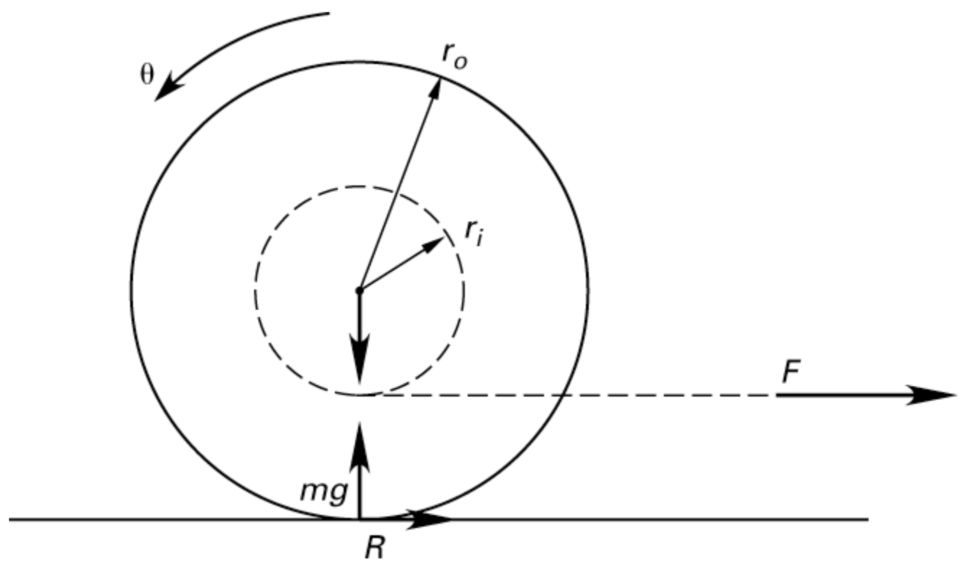
$$v_{max} = \pm 0.604 \text{ m/s}$$

$$a = v' = (-0.05 \text{ m})\left(12.08 \ \frac{\text{rad}}{\text{s}}\right)^2 \cos 12.08t$$

$$a_{max} = \pm 7.30 \text{ m/s}^2$$

Review

Example:



A yoyo (mass m, inertia I about center of gravity) is placed on a horizontal surface with a coefficient of friction μ. The string, wrapped underneath, is pulled with a force F. Determine if the yoyo will slip on the floor and which direction it will rotate as a function of F.

The two equations for the movement of the center of gravity and the rotation are

$$F + R = ma$$

$$Fr_i + Rr_o = I\alpha$$

Review

There are two possibilities for the third equation: slip or no slip.

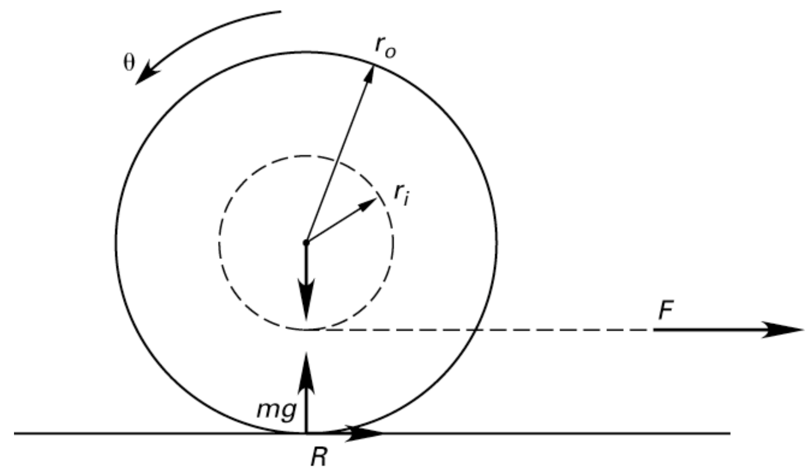
No slip: $|R| < \mu mg \Leftrightarrow a = -r_o\alpha$

Solving these 3 equations with 3 unknowns (α, a, and R) leads to

$$\alpha = -F\frac{r_o - r_i}{I + mr_o^2} \therefore \alpha < 0$$

$$a = \frac{Fr_o(r_o - r_i)}{I + mr_o^2} \therefore a > 0$$

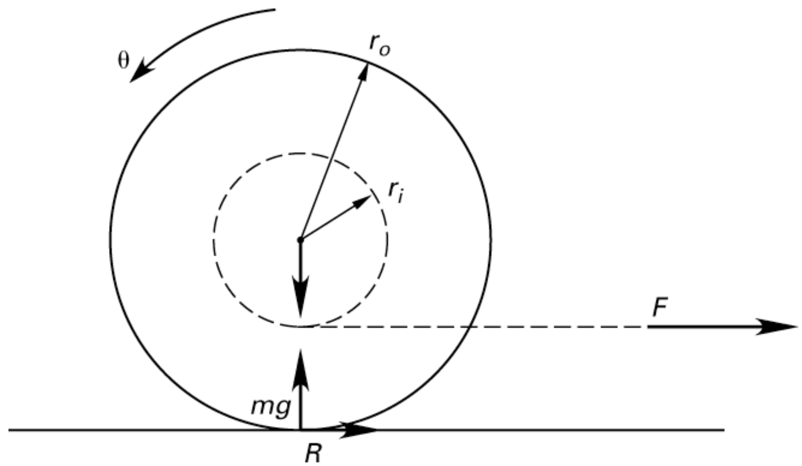
$$R = -F\frac{I + mr_i^2}{I + mr_o^2}$$



The yoyo is moving to the right while wrapping itself on the string. The solution for R gives the condition for no slip.

$$F < \mu mg\frac{I + mr_o^2}{I + mr_i^2}$$

Review



Slip: $R = -\mu mg$

Solving these 3 equations lead to

$$a = \frac{F - \mu mg}{m}$$

$$= \frac{Fr_i - \mu mgr_o}{I}$$

The yoyo always moves to the right, but can rotate forward or backard depending on the value for *F*.