USING SETS OF WINNING COALITIONS TO GENERATE FEASIBLE BANZHAF POWER DISTRIBUTIONS

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Abstract. In his paper, John Tolle enumerates the possible Banzhaf power distributions in a 4-player weighted voting system [8]. Expanding on Tolle's ideas, we construct sets of winning coalitions in the n-player system by organizing them into a rooted tree, utilizing the partially ordered lattices of weighted coalitions. By counting the nodes of the tree, we enumerate all possible sets of winning coalitions and all possible Banzhaf power distributions. We characterize these distributions by identifying the possible denominators and the necessary conditions on the numerators.

1. Introduction

It's not the voting that's democracy; it's the counting.
Tom Stoppard, *Jumpers*

From the founding fathers' debates over what constitutes fair representation to discussions of the Electoral College's effectiveness in recent presidential elections, the amount of power which individual voters and blocs of voters should receive has held a prominent position in American political thought.

This debate has inspired collaboration between the political and mathematical sciences. As mathematicians model voting systems, they often measure the concept of voting *power* as the potential of a *player* (a voting party who must cast all of its votes for or against an issue) to cast the deciding vote. In 1954 Lloyd Shapley and Martin Shubik [6] introduced the Shapley-Shubik power index, which considers all permutations of the players and determines the unique player in each grouping which changes the outcome of the vote. In the power distribution, a player's power is proportional to the number of permutations for which adding that player to a group causes that group to meet the quota.

In voting systems where the order of voting is not a factor, the *Banzhaf Power Index* (BPI) provides an alternative for measuring the power of a single player. In 1965, John C. Banzhaf III used the BPI to show that his Long Island district had no power in the County Board even though it controlled votes in proportion to its population [1]. Since then, the power index has been used to evaluate the distribution of power in systems such as the Electoral College of the United States, in which it appears that the smallest states control a disproportionate amount of power.

1.1. **Weighted Voting Systems.** A *weighted voting system*, $[q; v_1, v_2, \ldots, v_n]$, is defined as a collection of weighted players together with a *quota*, which is the total number of votes required to pass a motion. In a weighted voting system (WVS), a player's *weight* refers to the number of votes allotted to that player and is always a positive integer value. We denote a player by P_i and his weight by v_i. Players are ordered according to their weight, which means $v_1 \geq v_2 \geq \cdots \geq v_n$. The quota must be an integer satisfying the following criteria:

$$\frac{v_1 + v_2 + \cdots + v_n}{2} < q \leq v_1 + v_2 + \cdots + v_n. \tag{1}$$

A group of k players who vote as a bloc form a *k-coalition*. A coalition is ranked by the sum of the weights of the players contained in the coalition, an idea which will be discussed at length in Section 5. For now we need only consider the following definitions: the k-coalition $C = \{P_{i_1} \ldots P_{i_k}\}$ is a *winning coalition* if $v_c = v_{i_1} + \cdots + v_{i_k} \geq q$ and is *losing* if $v_{i_1} + \cdots + v_{i_k} < q$.

If a coalition C is winning, it must be true that $v_c \geq q$, which, together with (1) implies that $C^{\mathfrak{C}}$, the complement of C, is losing. For brevity's sake, we refer to this as the *complement rule*. Notice that the converse is not true since the complement of a losing coalition is not necessarily winning. As evidence, consider the 6-player system $[13; 5, 4, 3, 3, 2, 1]$ in which neither $\{P_1 P_2 P_3\}$ nor its complement $\{P_4 P_5 P_6\}$ is winning.

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We define a *supercoalition* of a coalition C to be a coalition that contains C. The set of supercoalitions of C is denoted $\mathbf{S(C)}$. If C is winning, it follows that so are all of its supercoalitions.

Another important characteristic of all weighted voting systems which we will consider is that for all i, $v_i < q$. That is, there are no one-player winning coalitions. This implies that no player is a dictator. Furthermore, for all i

$$\sum_{j \neq i} v_j \geq q, \tag{2}$$

so all $(n-1)$-coalitions are winning; i.e., no player has veto power.

1.2. **The Banzhaf Power Index.** Suppose the k-coalition $\{P_{i_1} \dots P_{i_k}\}$ is winning. We say that a member of the coalition is a *critical player* if the coalition loses when that player is removed. So, in the 6-player system $[13; 5, 4, 3, 3, 2, 1]$, $\{P_1P_2P_3P_4\}$ is a winning coalition where all players are critical.

A player's *Banzhaf Power Index*, denoted $B(P_i)$, is the ratio of the number of instances in which P_i is critical to the total number of critical instances. The *Banzhaf power distribution*, β, for an n-player weighted voting system is the vector $(B(P_1), \dots, B(P_n))$, the sum of whose components is one.

In order to calculate the Banzhaf power distribution of an n-player voting game, we first determine the set of winning coalitions (**WC**) in that game. For example, the 5-player voting game

$$[16; 6, 6, 5, 5, 3]$$

generates the following set of winning coalitions and critical instances:

Winning Coalition	Critical Players
$\{P_1P_2P_3P_4\}$	none
$\{P_1P_2P_3P_5\}$	P_1, P_2, P_3
$\{P_1P_2P_4P_5\}$	P_1, P_2, P_4
$\{P_1P_3P_4P_5\}$	P_1, P_3, P_4
$\{P_2P_3P_4P_5\}$	P_2, P_3, P_4
$\{P_1P_2P_3\}$	P_1, P_2, P_3
$\{P_1P_2P_4\}$	P_1, P_2, P_4
$\{P_1P_3P_4\}$	P_1, P_3, P_4
$\{P_2P_3P_4\}$	P_2, P_3, P_4

Using this table, we find that P_1, P_2, P_3 and P_4 are each critical 6 times, and P_5 is never critical. The Banzhaf power distribution for this voting game is

$$\beta = \left(\frac{6}{24}, \frac{6}{24}, \frac{6}{24}, \frac{6}{24}, 0\right)$$

Throughout this paper, we leave the fractions of the Banzhaf power distribution in their unreduced form, so that the denominator represents the total number of critical instances for that voting game.

1.3. **An Alternative Approach.** A voting game is traditionally described as a quota together with n weighted players. Our discussion utilizes a second representation, which classifies a voting game according to the set of winning coalitions it generates. Two voting games with different quotas and weights that generate the same set of winning coalitions are equivalent because this set of winning coalitions determines the power distribution of the voting game.

Example 1.1. Recall the set of winning coalitions generated by the 5-player voting game $[16; 6, 6, 5, 5, 3]$. Now consider the voting game $[31; 15, 12, 10, 9, 3]$. The reader can verify that these voting games generate the same set of winning coalitions, and are therefore equivalent.

1.4. **Motivation and Research Problem.** Mathematicians such as Klinz and Woeginger [4] and Tannenbaum [7] have studied the Banzhaf power distribution from a computational point of view. That is, given a voting game, how can the corresponding power distribution be most efficiently calculated? Tolle *et al.* have enumerated the possible Banzhaf power distributions for the 3-, 4-, and 5-player systems [8], [3]. Techniques outlined in these papers gave rise to our research, which focuses on enumerating and restricting possible Banzhaf power distributions.

We first calculated all possible power distributions for the 6-player system (see Appendix) and in this paper we generalize that method into an algorithm for counting possible Banzhaf power distributions in the n-player system. In addition, we placed restrictions on the possible power distributions, using properties that became evident as we developed the algorithm.

In this paper we first describe our method for constructing sets of winning coalitions. By considering both the critical instances of a system and of each player, this method allows us to place restrictions on possible Banzhaf power distributions. We then use this same method, together with the known relationship between the weighted coalitions, to develop an algorithm which enumerates all possible distributions.

2. Sets of Winning Coalitions

Before determining how many and which Banzhaf power distributions are possible in an n-player WVS, it is necessary to re-examine what is meant by *a set of winning coalitions*. It seems natural to build a set of winning coalitions by starting with the smallest-sized coalitions. For example, in a 6-player system, we may choose a set of winning coalitions that contains zero 2-coalitions, four 3-coalitions, all of their supercoalitions, and an additional 4-coalition. Of course, all 5-coalitions are winning since our voting systems have no veto power. Let us illustrate the bottom-up construction of this set of winning coalitions:

$$\begin{array}{llllll}
\{P_1P_2P_3\} & \Rightarrow & \{P_1P_2P_3P_4\} & \bigcup & \{P_1P_2P_3P_4P_5\} \\
 & & \{P_1P_2P_3P_5\} & & \{P_1P_2P_3P_4P_6\} \\
 & & \{P_1P_2P_3P_6\} & & \{P_1P_2P_3P_5P_6\} \\
\{P_1P_2P_4\} & \Rightarrow & \{P_1P_2P_4P_5\} & & \{P_1P_2P_4P_5P_6\} \\
 & & \{P_1P_2P_4P_6\} & & \{P_1P_3P_4P_5P_6\} \\
\{P_1P_3P_4\} & \Rightarrow & \{P_1P_3P_4P_5\} & & \{P_2P_3P_4P_5P_6\} \\
 & & \{P_1P_3P_4P_6\} & & \\
\{P_2P_3P_4\} & \Rightarrow & \{P_2P_3P_4P_5\} & & \\
 & & \{P_2P_3P_4P_6\} & & \\
 & + & \{P_1P_2P_5P_6\} & &
\end{array}$$

Alternatively, we could build the same set of winning coalitions from the top-down; that is, by starting with the set of $(n-1)$-coalitions (all of which are winning), and sequentially adding winning coalitions of smaller order. One important fact is that for any coalition $\mathbf{C}$, $\mathbf{C} \in \mathbf{WC}$ only if $\mathbf{S}(\mathbf{C}) \subset \mathbf{WC}$. In the above example, since all 5-coalitions are contained in $\mathbf{WC}$, all 4-coalitions *could* be added to $\mathbf{WC}$; only 10 were. Similarly, the subset of 3-coalitions whose supercoalitions of order 4 have all been added to $\mathbf{WC}$ *could* all be added themselves; our set of winning coalitions includes 3 of them. This process can be generalized for an n-player system by adding coalitions of smaller order until the desired set of winning coalitions has been attained. For example, the above set of winning coalitions is constructed in the following manner:

$$\begin{array}{lllll}
\{P_1P_2P_3P_4P_5\} & \bigcup & \{P_1P_2P_3P_4\} & & \\
\{P_1P_2P_3P_4P_6\} & & \{P_1P_2P_3P_5\} & & \\
\{P_1P_2P_3P_5P_6\} & & \{P_1P_2P_3P_6\} & \hookrightarrow & \{P_1P_2P_3\} \\
\{P_1P_2P_4P_5P_6\} & & \{P_1P_2P_4P_5\} & & \\
\{P_1P_3P_4P_5P_6\} & & \{P_1P_2P_4P_6\} & \hookrightarrow & \{P_1P_2P_4\} \\
\{P_2P_3P_4P_5P_6\} & & \{P_1P_3P_4P_5\} & & \\
 & & \{P_1P_3P_4P_6\} & \hookrightarrow & \{P_1P_3P_4\} \\
 & & \{P_2P_3P_4P_5\} & & \\
 & & \{P_2P_3P_4P_6\} & \hookrightarrow & \{P_2P_3P_4\} \\
 & & \{P_1P_2P_5P_6\} & &
\end{array}$$

Notice that $\{P_1P_2P_5\}$ was not added to $\mathbf{WC}$ even though all of its supercoalitions are winning. This represents a choice made when sets of winning coalitions are built. Including $\{P_1P_2P_5\}$ as a winning coalition would result in a new set of winning coalitions with a new Banzhaf power distribution.

At this point it is worth noting that when $k \leq n/2$, not all k-coalitions can be added to $\mathbf{WC}$ even if all of their supercoalitions are already contained. To illustrate this, consider a 5-player system with a set of winning coalitions containing all 4-coalitions and all 3-coalitions. Although all supercoalitions of $\{P_1P_2\}$ and $\{P_3P_4\}$ are contained in $\mathbf{WC}$, $\{P_1P_2\}$ and $\{P_3P_4\}$ cannot both win since $\{P_3P_4\}$ is contained in the complement of $\{P_1P_2\}$. We generalize this idea in the following theorem.

Theorem 2.1. *In an n-player voting system, for $k \leq \frac{n}{2}$, the maximum number of winning $k-$coalitions is $\binom{n-1}{k-1}$.*

Proof. Recall that if a $k-$coalition is winning, then its $(n-k)$-player complement is losing. This implies that two coalitions whose intersection is the empty set cannot both be winning.

It follows that the maximum number of $k-$coalitions that can all be winning is equal to the number of $k-$coalitions whose pairwise intersection is nonempty. Let M denote the maximal set of winning $k-$coalitions. Then $M = \{C_1, C_2, \ldots, C_m\}$ such that $\forall i, j \in \{1, \ldots, m\}$, $C_i \cap C_j \neq \varnothing$. According to the Erdős-Ko-Rado Theorem [5], $m = \binom{n-1}{k-1}$. Thus this is also the maximum number of winning $k-$coalitions when $k \leq \frac{n}{2}$. When $k > n/2$, all k-coalitions can win without violating the complement rule. $\square$

Summarized, this discussion of winning coalitions yield three facts that will become important. For every coalition $C \in \mathbf{WC}$,

(1) $\mathbf{S(C)} \subset \mathbf{WC}$,
(2) If $v_{C^*} > v_C$, then $C^* \in \mathbf{WC}$, and
(3) $C^{\mathrm{c}} \notin \mathbf{WC}$.

Rule (2) will be expanded upon in Section 5.

2.1. **Counting Critical Instances.** Since the number of critical instances is equal to the denominator of the unreduced Banzhaf Power Index, we would like to find a method for counting the number of critical instances in a set of winning coalitions. We will first consider how the number of critical instances changes as coalitions of smaller order are added to $\mathbf{WC}$.

Using our method of building $\mathbf{WC}$ from the top down, a k-coalition can be winning only if all of its supercoalitions are already contained in $\mathbf{WC}$. Adding a k-coalition to $\mathbf{WC}$ results in k additional critical instances since no coalitions with fewer than k members are winning, implying that each member in the k-coalition is critical. Now we examine the change in critical instances among the winning $(k+1)$-coalitions. Exactly $n-k$ coalitions of order $k+1$ contain the new k-coalition. In each of these coalitions, one critical instance is subtracted since there is one player who is not a member of the new winning k-coalition. Thus, the net change in critical instances when a k-coalition is added to $\mathbf{WC}$ is $k-(n-k)$.

The reader may be curious as to why we only examine the k- and $(k+1)$-coalitions to determine the change in critical instances. We propose the following:

Theorem 2.2. *Adding a k-coalition to* $\mathbf{WC}$ *changes only the number of critical instances contained in winning k-coalitions and winning $(k+1)$-coalitions.*

Proof. Consider the $(k+2)$-coalitions contained in $\mathbf{WC}$. Critical players in these coalitions are those players without whom the coalition becomes a losing $(k+1)$-coalition. Notice that we have not added or removed any $(k+1)$-coalitions from $\mathbf{WC}$ since a coalition is added to $\mathbf{WC}$ only if all of its supercoalitions are already contained in $\mathbf{WC}$. Thus we are never forced to add $(k+1)-, (k+2)-, \ldots, (n-2)-$ coalitions to $\mathbf{WC}$ upon adding a k-coalition. Since the set of winning $(k+1)$-coalitions remains the same, so do the critical instances among the set of winning $(k+2)$-coalitions. So we see that adding a k-coalition to $\mathbf{WC}$ changes only the number of critical instances in the winning k-coalitions and in the winning $(k+1)$-coalitions containing that k-coalition. $\square$

We use this important result to systematically examine the total number of critical instances in a set of winning coalitions by looking at the number of winning coalitions of each possible order. The same result allows us to describe the way in which the total number of critical instances changes when k-coalitions are added to the set of winning coalitions.

3. Preliminary Results

This method of constructing sets of winning coalitions and tracking the change in critical instances will eventually allow us to algorithmically enumerate all possible Banzhaf power distributions on n players. However, before developing the general algorithm, we generated all possible power distributions on 6 players (See Appendix). In calculating these power distributions, several properties of weighted voting systems became evident. Although the following theorems do not directly address our main research questions, they do provide insight into important properties of weighted voting systems.

3.1. **Accumulation of Power.** Although one basic property of our WVS is that no player can have dictatorial power (for all $i \in \{1, 2, \ldots, n\}$, $B(P_i) \neq 1$), it is not trivial to examine how much power one player can control. It has been shown that in the 4-player system, P_1 can control up to $\frac{1}{2}$ of the power, and in the 5-player system, P_1 controls up to $\frac{7}{11}$ of the total power. Our data reveals that in the 6-player system, P_1 controls up to $\frac{3}{4}$ of the total power. In an unpublished paper [3], Gay, Harris, and Tolle present the following theorem and proof on the potential of one player to accumulate power. Let M_n represent the maximum possible value of $B(P_1)$.

Theorem 3.1.

$$\lim_{x\to a} M_n = 1$$

Proof. Given n, consider the WVS $[n-1; n-2, 1, 1, \dots, 1]$. There are a total of $2n-3$ votes, and the quota is greater than half this total. We note that P_1 belongs to every winning coalition except

$$\{P_2P_3 \dots P_n\}$$

and moreover, P_1 is critical in every winning coalition to which it belongs except the n-player coalition. We now undertake to count these coalitions: There are $n-1$ winning 2-coalitions; they are

$$\{P_1P_2\}, \{P_1P_3\}, \dots, \{P_1P_n\}.$$

Noting that each of these contains P_1 and one other player, we have $\binom{n-1}{1}$ such coalitions.

The winning 3-coalitions number $\binom{n-1}{2}$, since each contains P_1 and two other players. Similarly, each winning 4-coalition contains P_1 and three other players so that these number $\binom{n-1}{3}$. We continue in this manner, finally considering the $(n-1)$-coalitions, which must all win; there are n of these, and P_1 belongs to $n-1$ of them. Each contains P_1 and $n-2$ other players, so that we can describe the number of such coalitions by $\binom{n-1}{n-2}$. So the total number of coalitions in which P_1 is critical is

$$\alpha = \sum_{k=1}^{n-2} \binom{n-1}{k}.$$

Now we must compute the total number of critical instances; for this, we simply determine how players other than P_1 may be critical. Choose P_i with $i \neq 1$, and note that $\{P_1P_i\}$ is a winning coalitions with both players critical. So this contributes one critical instance for P_i. Next consider the coalition $C = \{P_2P_3 \dots P_n\}$. This coalition wins, and P_i is critical, since C is the only winning coalition not containing P_1. (So if P_i is removed from C, the result is a losing coalition by virtue of $P_1 \notin C - \{P_i\}$.)

So far we have 2 critical instances for P_i. But there can be no more, because if $3 \leq k \leq n-2$, and P_i belongs to a winning k-coalition, then P_1 also belongs to this coalition, so removing P_i results in a $(k-1)$-coalition to which P_1 also belongs, yielding another winning coalition. Hence P_i cannot be critical in the k-coalition.

So P_1 is critical in α instances, and each other player is critical in 2 instances. The total number of critical instances is therefore $\alpha + 2(n-1)$, so that

$$B(P_i) = \frac{\alpha}{\alpha + 2(n-1)}.$$

We note that α is a polynomial in n, with degree greater than or equal to 2 whenever $n \geq 5$. Since

$$\lim_{n\to\infty} \frac{2(n-1)}{\alpha} = 0,$$

and since $M_n \geq B(P_1)$, we obtain the result. □

Initially it would be natural to assume that P_1 is more likely to control more power when there are powerless players in the system. If adding more powerless players to the n-player system always increased $B(P_1)$, then the maximal $B(P_1)$ in any system would be $\frac{1}{3}$, which is $B(P_1)$ in the 3-player system. We will show in the following theorem that the maximal $B(P_1)$ in the $(n+1)$-player system with one powerless player is the same as the maximal $B(P_1)$ in the n-player system.

Theorem 3.2. *Given the voting game $[q; v_1, \dots, v_{n-k}]$ with power distribution β, there exists a voting game with k powerless players and power distribution $\beta' = (\beta, 0_1, \dots, 0_k)$, for all $k \in \mathbb{Z}^+$.*

Proof. Given the system $V = [q; v_1, \dots, v_{n-k}]$ with power distribution β, the system $aV = [aq; av_1, \dots, av_{n-k}]$ where $a > 0$ also has power distribution β since for any winning coalition $\{P_{i_1} \dots P_{i_j}\}$ in V,

$$v_{i_1} + \dots + v_{i_j} \geq q \Leftrightarrow av_{i_1} + \dots + av_{i_j} \geq aq$$

Similarly, a losing coalition of voters in V is also a losing coalition in aV.

Now consider the system $V' = [(k+1)q; (k+1)v_1, \dots, (k+1)v_{n-k}, 1_1, \dots, 1_k]$. Let C_L be a losing coalition comprised of elements from $\{P_1, \dots, P_{n-k}\}$ with weight $(k+1)l$ for some $l \in \mathbb{Z}^+$ and $l < q$. Notice that

$$(k+1)q - (k+1)l = (k+1)(q-l) > k$$

and thus $\{C_L P_{n-k+1} \dots P_n\}$ is not a winning coalition. Similarly, if C_W is a winning coalition comprised of elements from $\{P_1, \dots, P_{n-k}\}$ with weight $(k+1)w$ for some $w \in \mathbb{Z}^+$ and $w > q$, then

$$(k+1)w - (k+1)q = (k+1)(w-q) > k$$

and therefore $P_{i_p} \in \{P_{n-k}, \dots, P_n\}$ is not critical in the winning coalition $\{C_W P_{n-k+1} \dots P_n\}$. Thus P_{i_p} is never critical, which means that $B(P_{i_p}) = 0$. Furthermore, we showed above that winning/losing coalitions in V remain winning/losing coalitions in V'. Thus V' has power distribution $(\beta, 0_1, \dots, 0_k)$. □

Because power distributions in the n-player system can be transferred to any larger sized system, we already know many of the possible power distributions for a system of a given size. In particular, we know that each uniform distribution in the n-player system corresponds to a distribution in the $(n+1)$-player system with all power uniformly distributed across the first n players. Next we examine the conditions under which power is uniformly distributed among n-players.

3.2. **Uniform Banzhaf Distributions.**

Lemma 3.3. *In an n-player WVS, if $\{P_{i_1} P_{i_2} \dots P_{i_k}\}$ is a losing coalition, then $\{P_j P_{i_2} \dots P_{i_k}\}$ where $j \notin \{i_1, i_2, \dots, i_k\}$ and $j > i_1$ is also a losing coalition.*

Proof. Let $C_L = \{P_{i_1} P_{i_2} \dots P_{i_k}\}$. Denote the weight of C_L by $v_{i_1} + v_c$ where v_c is the weight of $C_L - P_{i_1}$. Then $v_j + v_c \leq v_{i_1} + v_c$. Therefore C_L remains a losing coalition after P_{i_1} has been replaced by P_j. □

Lemma 3.4. *In an n-player WVS, if $\{P_{i_1} P_{i_2} \dots P_{i_k}\}$ is a winning coalition, then replacing P_{i_j} by P_b for some $j \in \{1, 2, \dots, k\}$, where $b \notin \{i_1, i_2, \dots, i_k\}$ and $b < i_j$, also yields a winning coalition.*

Proof. Given that $\{P_{i_1} P_{i_2} \dots P_{i_k}\}$ is a winning coalition in a voting game with quota q, $v_{i_1} + v_{i_2} + \cdots + v_{i_k} \geq q$. It is easily observed that replacing v_{i_j} by v_b where $b < i_j$ also yields a winning coalition. Therefore, the coalition remains winning after any one of its members is replaced by a member of greater or equal weight. □

Together, these lemmata imply that P_i has at least as many critical instances as P_j where $j > i$. We will use this fact to prove the following result about uniform Banzhaf power distributions:

Theorem 3.5. *In an n-player voting system, the Banzhaf power distribution β is uniform if and only if there exists an integer k such that all k-coalitions are winning, and for all $j < k$, all j-coalitions are losing.*

Proof. First consider an n-player voting system in which all k-coalitions are winning and all j-coalitions, $j < k$, are losing. Then there are $\binom{n}{k}$ winning k-coalitions. Since none of the $(k-1)$-coalitions are winning, each k-coalition has k critical instances. Furthermore, each of the $(k+1)-, (k+2)-, \dots, n-$coalitions contains zero critical instances because each is a supercoalition of a winning k-coalition.

Thus each player is critical in exactly the number of k-coalitions in which it appears, which is the same as the number of $(k-1)$-coalitions that can be formed with the remaining $n-1$ players. This results in a total of $k \cdot \binom{n}{k}$ critical instances, where each P_i, $i = 1, 2, \dots, n$ is critical $\binom{n-1}{k-1}$ times. So for all i

$$B(P_i) = \frac{\binom{n-1}{k-1}}{k \cdot \binom{n}{k}}$$

Therefore β is uniformly distributed.

It remains to show that a uniform Banzhaf power distribution implies that there exists an integer k such that all k-coalitions are winning, and for all $j < k$, all j-coalitions are losing. Let k be the smallest integer for which there exists a winning k-coalition. Assume, to arrive at contradiction, that not all k-coalitions are winning. Suppose $C_W = \{P_{i_1} \dots P_{i_k}\}$ is the highest-weighted winning k-coalition. We know that P_{i_1} is critical since there are no winning $(k-1)$-coalitions. Now consider the k-coalition $C_{j_1} = \{P_{j_1} P_{i_2} \dots P_{i_k}\}$ where $j_1 \notin \{i_1, \dots, i_k\}$ and $j_1 > i_1$.

By Lemmata 3.3 and 3.4, P_{i_1} has at least as many critical instances as P_{j_1}; that is, $C(P_{i_1}) \geq C(P_{j_1})$.

We return our attention to the $k-$coalition C_{j_1}. If C_{j_1} is losing, then it contains no critical instances and $C(P_{j_1}) < C(P_{i_1})$, leading to a contradiction. If C_{j_1} is winning, then P_{j_1} is critical and we continue by replacing P_{i_1} with P_{j_2} where $j_2 \notin \{i_1, \dots, i_k\}$ and $j_2 > j_1$. Call the resulting coalition C_{j_2}. If C_{j_2} is losing, then $C(P_{j_2}) < C(P_{i_1})$. If C_{j_2} is winning, then replace P_{i_1} with P_{j_3} and repeat.

Iterate the process by replacing P_{i_1} with each player of lesser weight until a resulting coalition is losing, and thus there is a player with fewer critical instances than P_{i_1}. If all of the resulting coalitions are winning, then repeat the process by replacing P_{i_2} with players of lesser weight. Again, repeat until either one resulting coalition is losing, in which case there is a player with fewer critical instances than P_{i_2}, or until all possible players have replaced P_{i_2} and all of the resulting coalitions are still winning. In this case, begin again with the winning k-coalition of next-highest weight.

If no losing coalition is found, then all $k-$coalitions are winning and we have reached a contradiction, since we assumed that not all $k-$coalitions are winning. If a losing coalition is found, then there are at least two players with distinct Banzhaf Power Indices and thus the power distribution is not uniform. □

Observe that in the 4-player system, the only possible value for k in this theorem is 3. In the 5-player system, the possible values of k are 3 and 4, and in the 6-player system, $k = 4$ or $k = 5$. Having categorized the sets of winning coalitions which generate the uniform Banzhaf power distribution, we explore the number of ways in which power can be uniformly distributed among n players.

Corollary 3.6. *In an n-player WVS, the uniform Banzhaf distribution is obtained in $\lceil \frac{n}{2} - 1 \rceil$ cases.*

Proof. By Theorem 3.5, the uniform Banzhaf distribution only occurs when there exists a smallest k such that all k-coalitions are winning and for all $j < k$, all j-coalitions are losing. By the complement rule of our weighted voting systems, for an integer $k \leq \frac{n}{2}$, not all k-coalitions can win. Together with the veto condition, this implies that precisely those values of k such that $n > k \geq \lceil n/2 \rceil$ yield the uniform Banzhaf power distribution. There are $\lceil \frac{n}{2} - 1 \rceil$ of these values. □

By Theorem 3.2 and Corollary 3.6, we limit the number of distinct power distributions on n players in the following result.

Corollary 3.7. *Let m denote the number of possible Banzhaf power distributions in a WVS on n-players where $n \geq 5$, and m_D denote the number of distinct Banzhaf power distributions. Then*

$$m_D \leq m - \lceil \frac{n}{2} - 2 \rceil - \lceil \frac{n-1}{2} - 2 \rceil - \ldots - \lceil \frac{5}{2} - 2 \rceil.$$

Proof. Using induction, consider the 5-player WVS as the base case. Gay, Harris, and Tolle have proven that the 5-player system has $m = 36$ Banzhaf power distributions [3]. By Corollary 3.6, $\lceil \frac{5}{2} - 1 \rceil = 2$ of these distributions reduce to the uniform power distribution $\beta = (\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})$. The two cases yield $\lceil \frac{5}{2} - 1 \rceil - 1$ distinct Banzhaf power distributions. Thus, the number of distinct Banzhaf power distributions in the 5-player system is 35 by inspection and

$$36 - \lceil \frac{5}{2} - 1 \rceil - 1 = 36 - (\lceil \frac{5}{2} - 1 \rceil - 1) = 36 - \lceil \frac{5}{2} - 2 \rceil \geq 35.$$

Let l denote the possible Banzhaf power distributions in a weighted voting system on $(n-1)$-players and l_D denote the number of distinct Banzhaf power distributions. Then

$$l_D \leq l - \lceil \frac{n-1}{2} - 2 \rceil - \lceil \frac{n-2}{2} - 2 \rceil - \ldots - \lceil \frac{5}{2} - 2 \rceil.$$

Now consider the m power distributions in the n-player weighted voting system. By Theorem 3.2, $m = l+q$, where $q \in \mathbb{Z}^+$ is the number of cases unique to the n-player system. Thus, the number of distinct cases in the n-player system is the sum of the number of distinct cases in the $(n-1)$-player system and the number of distinct cases unique to the n-player system: $m_D = l_D + q_D$. By Corollary 3.6, we know there are $\lceil \frac{n}{2} - 1 \rceil$ such distributions. Thus,

$$\begin{aligned} q_D &= q - \lceil \frac{n}{2} - 1 \rceil + 1 \\ &= (m - l) - \lceil \frac{n}{2} - 1 \rceil + 1 \end{aligned}$$

and $m_D = l_D + [(m-l) - \lceil \frac{n}{2} - 1 \rceil + 1]$. Thus, by our assumption,

$$\begin{aligned} m_D &\leq l - \lceil \frac{n-1}{2} - 2 \rceil - \lceil \frac{n-2}{2} - 2 \rceil - \ldots - \lceil \frac{5}{2} - 2 \rceil + [(m-l) - \lceil \frac{n}{2} - 1 \rceil + 1] \\ &= m - \lceil \frac{n}{2} - 2 \rceil - \lceil \frac{n-1}{2} - 2 \rceil - \ldots - \lceil \frac{5}{2} - 2 \rceil. \end{aligned}$$

□

The results presented in this section reinforce the connection between sets of winning coalitions and Banzhaf power distributions. In the remainder of the paper, we directly address the problem of restricting and enumerating the possible power distributions on n players.

4. Restricting Possible Banzhaf Power Distributions

Every set of winning coalitions includes the n- and $(n-1)$-coalitions. The set containing only these coalitions yields a known Banzhaf power distribution. We are interested in exploring the way in which this distribution changes as coalitions of smaller sizes are added to the set of winning coalitions. Our first result answers this question.

Theorem 4.1. *Let d denote the number of critical instances (i.e., the denominator of an unreduced Banzhaf power distribution) in an n-player system.*

1. *When $k > n/2$, d increases as k-coalitions are added to* **WC**.
2. *When $k < n/2$, d decreases as k-coalitions are added to* **WC**.
3. *When $k = n/2$, d stays constant as k-coalitions are added to* **WC**.

Proof. Recall from Theorem 2.2 that the number of winning $k-$coalitions alone determines the number of critical instances in winning $k+1$-coalitions. Each k-coalition is contained in exactly $n-k$ different $(k+1)$-coalitions. Thus, each winning k-coalition adds k critical instances (one for each player contained in the coalition) and eliminates $n-k$ critical instances (one for each player not contained in the winning k-coalition). We find that the net change in critical instances is $k-(n-k)$, and so d increases when $k > n/2$, decreases when $k < n/2$, and stays constant when $k = n/2$. □

To illustrate this result for $k < n/2$, consider the following example.

Example 4.2. Consider an 8-player system in which **WC** consists of all 7-, 6-, 5-, and 4-player coalitions. Now add $\{P_1P_2P_3\}$ to **WC**. Each of the three players is critical in this coalition. Now examine the winning 4-coalitions. $\{P_1P_2P_3\}$ is contained in 5 4-coalitions: $\{P_1P_2P_3P_4\}, \{P_1P_2P_3P_5\}, \{P_1P_2P_3P_6\}, \{P_1P_2P_3P_7\}$, and $\{P_1P_2P_3P_4\}$. Each of these 4-coalitions loses one critical instance (P_4, P_5, P_6, P_7, and P_8, respectively), resulting in a net loss of $3-5=2$ critical instances.

Now we can describe the way in which the total number of critical instances changes as coalitions are added to **WC**. Next we prove that sets of winning coalitions which are distinct but have the same structure also have the same number of critical instances.

Theorem 4.3. *In an n-player system, two sets of winning coalitions* **WC** *and* **WC**$'$ *with the same number of winning j-coalitions for $j \in \{2, 3, \cdots, n-1\} - \{n/2\}$ have the same number of critical instances.*

Proof. Before proving this inductively, note that the number of $\frac{n}{2}$-coalitions in **WC** and **WC**$'$ does not affect the total number of critical instance since each coalition adds $\frac{n}{2} - (n - \frac{n}{2}) = 0$ critical instances. Recall that **WC** and **WC**$'$ contain the same n-coalition and the same n $(n-1)$-coalitions. If neither set includes any coalitions of smaller order, then the total number of critical instances in each is $n(n-1)$. As a base case, suppose **WC** and **WC**$'$ each contain c_{n-2} $(n-2)$-coalitions and no coalitions of smaller order. Each $(n-2)$-coalition is contained in 2 $(n-1)$-coalitions, and therefore each results in a net change of $(n-2)-2$ critical instances. Therefore, **WC** and **WC**$'$ both contain $c_{n-2}(n-4)$ critical instances.

Now assume that if **WC** and **WC**$'$ contain the same number of $(n-2)-, \ldots, (n-k)$-coalitions and no coalitions of smaller order, then **WC** and **WC**$'$ contain the same number of critical instances. It remains to show that this is true when **WC** and **WC**$'$ contain the same number of $(n-2)-, \ldots, (n-k), (n-k-1)$-coalitions and no coalitions of smaller order.

By Theorem 2.2, we need only consider the change in critical instances in $\mathbf{WC_{n-k-1}}$ and $\mathbf{WC_{n-k}}$. Suppose **WC** and **WC**$'$ each contain c_{n-k-1} $(n-k-1)$-coalitions and no coalitions of smaller order. Each $(n-k-1)$-coalition is contained in $k+1$ $(n-k)$-coalitions, and therefore each $(n-k-1)$-coalition results in a net change of $n-k-1-(k+1) = n-2k-2$ critical instances. So, the change in critical instances in each set of winning coalitions is the same, and thus the total number of critical instances remains the same. □

In addition, we can calculate this number of critical instances given a set of winning coalitions.

Corollary 4.4. *A set of winning coalitions containing n $(n-1)$-coalitions, c_2 $(n-2)$-coalitions, c_3 $(n-3)$-coalitions, $\cdots$, c_{n-2} 2-coalitions contains*

$$n(n-1) + c_2((n-2) - (n-(n-2))) + c_3((n-3) - (n-(n-3))) + \cdots + c_{n-2}(2-(n-2))$$

$$= n(n-1) + c_2(n-4) + c_3(n-6) + \cdots + c_{n-2}(4-n)$$

critical instances.

The preceding results allow us to determine the denominator of the Banzhaf power distribution corresponding to a given set of winning coalitions. An equally powerful tool is the ability to bound this denominator knowing only the number of players in a voting game, but not the corresponding set of winning coalitions. We present this result in the next theorem.

Theorem 4.5. *In an n-player system,*

$$\binom{n}{n-1} \leq d \leq \binom{n}{\lceil n/2 \rceil} \lceil n/2 \rceil$$

where d denotes the total number of critical instances (i.e., the denominator of the unreduced Banzhaf power distribution) in a set of winning coalitions.

Proof. Since we assume that no player has veto power, all $(n-1)$-coalitions are winning in the n-player system. When **WC** is the set of all $(n-1)$-coalitions, the total number of critical instances is $\binom{n}{n-1}(n-1)$. Now we must show that this is the minimum number of critical instances for the system.

When $n > 4$, $n - 2 > n/2$, which means that the number of critical instances increases when an $(n-2)$-coalition is added to **WC**. By Theorem 4.1, the denominator increases whenever a coalition with more than $n/2$ players is added to **WC**. We also saw that the denominator decreases when a coalition with fewer than $n/2$ players is added to **WC**, which means we must prove that the denominator never drops below $\binom{n}{n-1}(n-1)$.

Also by Theorem 4.1, adding a coalition or order less than $\frac{n}{2}$ to the set of winning coalitions decreases the number of critical instances in a voting game. It would seem to follow that the voting game in which all possible coalitions of order less than $\frac{n}{2}$ are winning produces the least number of critical instances. However, we will show that the voting game in which only the $(n-1)$-coalitions are winning has even fewer critical instances.

Let us first consider the former case, i.e. suppose that *all* 2-coalitions are winning. The total number of critical instances, then, is $\binom{n}{2}(2)$. Expansion of the binomial coefficient reveals that

$$\begin{aligned} \binom{n}{2}(2) &= n(n-1) \\ \Rightarrow \binom{n}{2}(2) &= \binom{n}{n-1}(n-1). \end{aligned}$$

By Theorem 2.1, a maximum of $(n-1)$ 2-coalitions can be winning, and by Theorem 4.1, removing a 2-player coalition from **WC** causes a net increase in critical instances. Therefore, the number of critical instances when all *possible* 2-coalitions are winning is at least $\binom{n}{n-1}(n-1)$. So, when $n > 4$, the number of critical instances in an n-system is smallest when the $(n-1)$-coalitions are the only winning coalitions. When $n = 4$, the denominator of all possible Banzhaf power distributions is 12.

Now we seek to place an upper bound on the number of critical instances in an n-player system. From Theorem 4.1, we know that adding winning coalitions of size greater than $\frac{n}{2}$ always increases the total number of critical instances. It remains to show that the maximum number of critical instances is attained when all coalitions of size $\lceil \frac{n}{2} \rceil$ are winning. Recall that adding winning coalitions of order less than $\frac{n}{2}$ decreases the total number of critical instances. Thus the maximum denominator for an n-player system is

$$\binom{n}{\lceil \frac{n}{2} \rceil} \lceil \frac{n}{2} \rceil,$$

and occurs when all coalitions of order greater than $\frac{n}{2}$ are winning. □

In addition, our data supports the following conjecture.

Conjecture 4.6. *In an n-player system, the possible denominators of Banzhaf power distributions increase by 1 if n is odd, and by 2 if n is even.*

We have now shown that, for an n-player system, the minimum number of critical instances is attained when only the $(n-1)$-coalitions are winning and the maximum number of critical instances is attained when all coalitions of order greater than $\frac{n}{2}$ are winning. Knowing these bounds significantly reduces the number of possible power distributions on n players. We further restrict these possibilities by considering each player's Banzhaf Power Index, represented by the numerators of the Banzhaf power distribution.

Theorem 4.7. *In any Banzhaf power distribution, all numerators have the same parity.*

Proof. For every n, there exists a voting game on n players that results in a uniform Banzhaf power distribution. Specifically, the n-player voting game $[n-1; 1, 1, \ldots, 1]$ results in a set of winning coalitions consisting entirely of the $(n-1)$-coalitions; there are n of these. The total number of critical instances is $n(n-1)$, and each player is critical in exactly the number of $(n-1)$-coalitions in which it appears. Thus, for all $i \in \{1, 2, \cdots, n\}$, $B(P_i) = \frac{(n-1)}{n(n-1)}$.

According to our representation of sets of winning coalitions, adding a winning k-coalition has two effects. First, each player contained in the k-coalition gains one critical instance since no coalitions of smaller size are winning. Second, one critical instance is subtracted for each $(k+1)$-coalition containing the winning k-coalition. Specifically, one critical instance is subtracted for each player *not* included in the winning k-coalition. Thus adding a coalition to **WC** causes each player's power index to increase by one or decrease by one. Since there is a voting game for which all players have the same power index, all numerators will have the same parity in any Banzhaf power distribution. □

The preceding theorems place restrictions on which Banzhaf power distributions are possible for n players. The characteristics of power distributions described above became evident even when calculating distributions by hand. However, the method used in proving these theorems eventually led to the development of an algorithm, which enumerates all possible sets of winning coalitions. The remainder of this paper will address the other part of our other research question: In a WVS on n players, how many Banzhaf power distributions are possible? In order to answer this question, we first introduce a structure which will allow us to illustrate and organize the relationships between coalitions of a given size.

5. Using Inequality Lattices to Compare k-coalitions

5.1. **Defining the Inequality Lattices.** The set of n players in a WVS constitutes a partially ordered set of players ranked according to the weights of their votes: $v_1 \geq v_2 \geq \ldots \geq v_n$. A *partially ordered set* is a set in which $x \geq y$ is defined [2] such that the following properties are obeyed:

- *Reflexive*: For all x, $x \geq x$;
- *Antisymmetric*: If $x \geq y$ and $y \geq x$, then $x = y$;
- *Transitive*: If $x \geq y$ and $y \geq z$, then $x \geq z$.

Because players are ranked, coalitions of players are also ranked by the sum of the weights of their member players. Each set of k-coalitions defines a partially ordered set that may be represented as an inequality lattice.

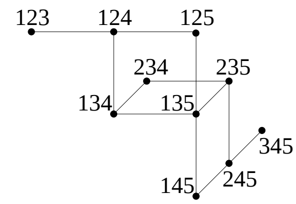
Example 5.1. Consider the inequality lattice that relates the weights of the ten 2-coalitions in the WVS on 5 players. We define this 2-dimensional structure as the 2-*coalition inequality lattice for the* 5-*player WVS*, which illustrates the relationships between all 2-coalitions in the 5-player WVS.

$$
\begin{array}{ccccccc}
(v_1+v_2) & \geq & (v_1+v_3) & \geq & (v_1+v_4) & \geq & (v_1+v_5) \\
 & & \geq & & \geq & & \geq \\
 & & (v_2+v_3) & \geq & (v_2+v_4) & \geq & (v_2+v_5) \\
 & & & & \geq & & \geq \\
 & & & & (v_3+v_4) & \geq & (v_3+v_5) \\
 & & & & & & \geq \\
 & & & & & & (v_4+v_5)
\end{array}
$$

Notice that in the above lattice, not all of the relationships between the coalition weights are illustrated expressly. For instance, $\{P_1P_2\} \geq \{P_4P_5\}$, which is indirectly illustrated by several different paths or *chains* of the lattice. This is because inequality lattices show a binary relationship between two k-coalitions, C_1 and C_2, only when C_1 covers C_2. We say that C_1 *covers* C_2 if $C_1 > C_2$ and there is no coalition C_x such that $C_1 \geq C_x \geq C_2$. The coalition which is not covered by any other k-coalition is the *root*, or maximally weighted k-coalition, $\{P_1, \ldots, P_k\}$, of the lattice. Thus the root of the above lattice is $\{P_1P_2\}$.

The lattice representation of the relative weights of the k-coalitions is helpful because it simultaneously illustrates all relationships between the weights of coalitions. The reader may verify that 1-coalitions form a simple 1-dimensional chain. With larger coalitions it is necessary for the inequality lattices to spread into the second dimension, such as in Example 5.1. Similarly, the 3-coalitions in the 5-player WVS necessitate a three-dimensional lattice to represent a greater number of relationships. The 3-dimensional inequality lattice for 3-coalitions in the 5-player WVS can also be represented graphically. Each coalition is represented by a vertex and the binary relationship $\geq$ between two coalitions is represented as an edge. The graphic lattice is read left to right, top to bottom, and front to back. Thus, $(v_1+v_2+v_3) \geq (v_1+v_2+v_4) \geq \ldots \geq (v_3+v_4+v_5)$.

Example 5.2. Depicted below is the 3-coalition inequality lattice in the 5-player WVS, with each player represented by its subscript:



Notice that $\{P_1P_2P_3\}$, the coalition furthest to the front left, is the coalition of highest weight and $\{P_3P_4P_5\}$, the coalition furthest to the rear right, is the coalition of least weight. These elements of the partially ordered set are *maximal* and *minimal* weighted coalitions, respectively. Notice that there are multiple inequality chains which connect the maximal and minimal weighted coalitions.

Using this method, we can describe the relative ranks of all k-coalitions in an n-player WVS using finite-dimensional inequality lattices. We now relate this discussion of inequality lattices back to our earlier discussion regarding the construction of sets of winning coalitions.

5.2. **Sets of Winning Coalitions from Lattices.** Recall the discussion in Section 2 of the rules governing the addition of k-coalitions to sets of winning coalitions. A k-coalition C may only be added to the set if it satisfies all of the following:

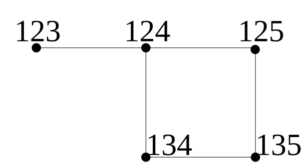
(1) $\mathbf{S(C)} \subset \mathbf{WC}$,
(2) If $v_{C^*} > v_C$, then $C^* \in \mathbf{WC}$, and
(3) $C^{\complement} \notin \mathbf{WC}$.

Before expanding upon rule (2) using inequality lattices, we present the following example.

Example 5.3. Suppose we wish to add $\{P_1P_3P_5\}$ to the set of winning 3-coalitions in a 5-player system. To do so, we will use the 3-coalition inequality lattice on 5-players illustrated in Example 5.2.

We know that the two coalitions which cover $\{P_1P_3P_5\}$, $\{P_1P_2P_5\}$ and $\{P_1P_3P_4\}$, must already be winning. It follows that $\{P_1P_2P_4\}$ must also already be winning because it covers both $\{P_1P_2P_5\}$ and $\{P_1P_3P_4\}$. Likewise, $\{P_1P_2P_3\}$ must be winning because it covers $\{P_1P_2P_4\}$. In this example, $\{P_1P_2P_3\}$ is the root.

This yields the following set of winning k-coalitions, with each player represented by its subscript:



We call the subset of five 3-coalitions from Example 5.3 a *rooted rectangular sublattice* of the 3-coalition inequality lattice on 5-players. A rooted rectangular k-coalition sublattice must always contain the maximal k-coalition of the lattice, that is $\{P_1P_2 \ldots P_k\}$. We call the sublattice *rectangular* because if the root and an additional k-coalition are taken to be winning, then so must all coalitions contained in the s-dimensional rectangle whose maximal diagonal has those two coalitions as endpoints. A set of winning k-coalitions must form a rooted rectangular sublattice of the k-coalition lattice for n-players.

With this additional means of describing the relationships between coalitions of a given size, we are ready to introduce a method which allows us to determine the possible sets of winning coalitions in an n-player system.

6. Using Rooted Trees to Describe All Sets of Winning Coalitions

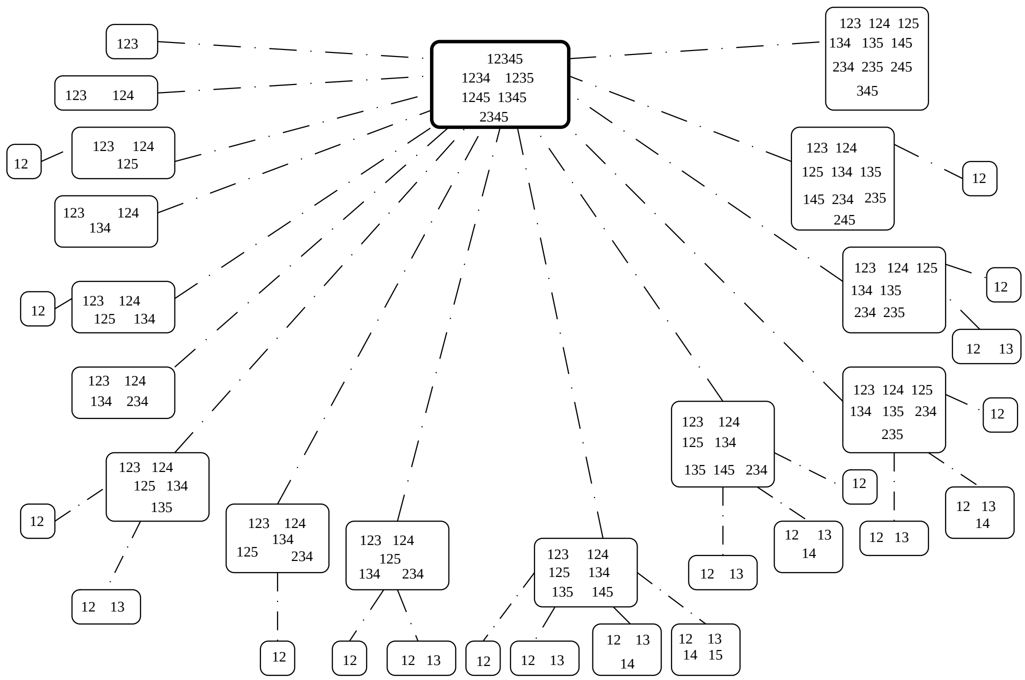
Inequality lattices allow us to individually construct all sets of winning coalitions in an n-player voting system. This information can be organized into a rooted tree on which each possible set of winning coalitions is described by a distinct path. The following algorithm describes a method for building this tree. When implemented, it allows us to count the number of distinct sets of winning coalitions by counting the number of leaves on the tree.

6.1. **To Build the Tree.** The root of the tree is formed by the set of n- and $(n-1)$-coalitions, all of which are required to win in a WVS. All possible sets of winning $(n-2)$-coalitions form the first generation of children, followed by winning $(n-3)$-coalitions. The final generation consists of sets of winning 2-coalitions.

To construct child nodes, we examine the inequality lattices discussed in Section 5. In order for a set of k-coalitions to constitute a child node, the following criteria must be met for every coalition C in the set:

(1) Each element of $\mathbf{S}(\mathbf{C})$ must appear in an ancestral node;
(2) The set of winning coalitions in the node forms a rooted rectangular sublattice;
(3) No ancestor of C contains $C^{\complement}$.

Example 6.1. Illustrated below is the rooted tree for the 5-player WVS, on which each player is represented by its subscript.



Taking a moment to count the number of nodes on tree, the reader may see that there are only 35 nodes, corresponding exactly to the number of power distributions on five players. Indeed, as we show with the next theorem, the n-player tree depicts all possible sets of winning coalitions in the n-player system.

Theorem 6.2. *There is a one-to-one correspondence between rooted paths on the tree and possible sets of winning coalitions.*

Proof. First we show that every path on the tree represents a set of winning coalitions. Recall the conditions on building constructing the rooted tree: if a coalition appears in a k^{th}-generation child, then its supercoalitions must appear in ancestor nodes; the set of coalitions contained in a child must form a rooted rectangular sublattice; and a coalition and its complement may not both appear along a rooted path. Notice from the discussion in Section 5 that any set of coalitions meeting these requirements is also valid set of winning coalitions. Therefore all rooted paths on the tree represent legitimate sets of winning coalitions.

To show that all sets of winning coalitions are represented on the tree, consider some set of winning coalitions, $\mathbf{WC}$, which is composed of subsets containing n-, $(n-1)$-, $\ldots$, or 2-coalitions.

First notice that the subsets containing the n- and $(n-1)$-coalitions must appear in $\mathbf{WC}$ since these subsets together correspond to the root node of the tree. If $\mathbf{WC}$ contains a subset of $(n-2)$-coalitions, then the elements of this subset must form a rooted rectangular sublattice. This subset corresponds to a first-generation node.

Any additional subset of $\mathbf{WC}$ containing $(n-k)$-coalitions must form a rooted rectangular sublattice. Additionally, all supercoalitions of each element in the subset must be contained in $\mathbf{WC}$. Finally, the addition of the subset must not force $\mathbf{WC}$ to violate the complement rule.

Thus, each subset of $\mathbf{WC}$ corresponds to a node along a rooted path of the tree, and every set of winning coalitions appears as a rooted path. $\square$

Corollary 6.3. *Because every rooted path on the tree represents a properly constructed set of winning coalitions, the number of possible power distributions in the n-player system is the same as the number of nodes on the rooted tree representing that system.*

This corollary allows us to count the number of possible power distributions in the n-player system, answering one of our research questions. Furthermore, our data suggests that distinct power distributions correspond to distinct sets of winning coalitions, which we consider in the following section.

7. Conjectures on Distinct Power Distributions

In this section we consider Banzhaf power distributions in their unreduced form. For example, we refer to

$$\left(\tfrac{10}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{10}{60}\right) \text{ and } \left(\tfrac{5}{30}, \tfrac{5}{30}, \tfrac{5}{30}, \tfrac{5}{30}, \tfrac{5}{30}, \tfrac{5}{30}\right)$$

as distinct because they represent different numbers of critical instances.

Let $\mathbf{WC}$ and $\mathbf{WC}'$ represent distinct sets of winning coalitions with corresponding power distributions $\beta = (B(P_1), \ldots, B(P_n))$ and $\beta' = (B'(P_1), \ldots, B'(P_n))$. We let j denote the largest sized coalition for which $\mathbf{WC_j} \neq \mathbf{WC'_j}$ where $\mathbf{WC_j}$ and $\mathbf{WC'_j}$ represent the j-coalitions contained in $\mathbf{WC}$ and $\mathbf{WC}'$, respectively.

Theorem 7.1. *If $|\mathbf{WC_j}| \neq |\mathbf{WC'_j}|$ and $\mathbf{WC_m} = \mathbf{WC'_m}$ for all $m \neq j$, then $\beta \neq \beta'$.*

Proof. Let $|\mathbf{WC_j}| = A$ and $\mathbf{WC'_j} = B$ where $A \neq B$. The total number of critical instances in $\mathbf{WC} - \mathbf{WC_j}$ is equal to the total number of critical instances in $\mathbf{WC}' - \mathbf{WC'_j}$; call this number N. Since each j-coalition changes the number of critical instances in a set of winning coalitions by $2j - n$, the number of critical instances in $\mathbf{WC}$ is equal to $N + A(2j - n)$, while the number of critical instances in $\mathbf{WC}'$ is equal to $N + B(2j - n)$. Thus there exists at least one player P_i for whom $B(P_i) \neq B'(P_i)$, which implies that $\beta \neq \beta'$. □

Conjecture 7.2. *Distinct sets of winning coalitions correspond to distinct unreduced Banzhaf power distributions.*

Thus far, the n-player rooted tree allows us to enumerate all possible distinct sets of winning coalitions. A proof of Conjecture 7.2 would allow us to assign each of these sets a distinct power distribution, enabling us to count the number of distinct Banzhaf power distributions on n players directly from the rooted tree.

8. Open Questions

In addition to Conjectures 4.6 and 7.2, we are left with several queries, which may warrant further research. The theorems presented in Sections 3 and 4 place restrictions on the possible Banzhaf power distributions for n players. While these significantly limit the possible power distributions, we are not yet able to determine the valididty of a given power distribution. A complete set of restrictions would allow us to determine whether or not a power distribution represents a feasible voting game, answering a question initially posed by John Tolle [8].

We have outlined a method for generating the rooted tree for the n-player system, on which each node corresponds to a distinct set of winning coalitions and Banzhaf power distribution. However, as n increases, the number of power distributions increases exponentially. Thus, listing all sets of winning coalitions becomes computationally inefficient as the size of the system increases. In light of this, it would be useful to develop a closed-form formula which yields the same number of power distributions.

The rooted tree describes sets of winning coalitions in the n-player system, which in turn determine Banzhaf power distributions. For practical purposes, it would be useful to create an algorithm which generates a voting game from a feasible set of winning coalitions. Furthermore, is it possible to determine the set of winning coalitions that corresponds to a known power distribution?

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Appendix A. The Banzhaf Power Distributions of a 6-Player Weighted Voting System

We constructed the 6-player tree using the method outlined in our paper, thus generating all 446 sets of winning coalitions for the WVS. This appendix details those cases. Each case is read horizontally, beginning with the unique alphanumeric label of the form $n_2.n_3.n_4$ appearing to the left of the entry, where each n_i indicates the number of i-coalitions in the set. Letters appearing to the right of the n_i indicate that there are multiple ways of including that number of i-coalitions in the set.

Example A.1. The label $1.4.9b$ means that this case is defined by 1 winning 2-coalition, 4 winning 3-coalitions, and 9 winning 4-coalitions. However, b denotes that this is only one of the ways that 9 4-coalitions may win given this particular set of winning 2- and 3-coalitions.

The label is followed by the corresponding set of winning coalitions and Banzhaf power distribution.

0.0.0	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_5P_6$	$\beta = (\frac{5}{30}, \frac{5}{30}, \frac{5}{30}, \frac{5}{30}, \frac{5}{30}, \frac{5}{30})$
	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4P_5P_6$	
	$P_2P_3P_4P_5P_6$	

0.0.1	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_3P_4P_6$	
		$P_1P_2P_3P_5P_6$	$\beta = (\frac{6}{32}, \frac{6}{32}, \frac{6}{32}, \frac{6}{32}, \frac{4}{32}, \frac{4}{32})$
		$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4P_5P_6$	
		$P_2P_3P_4P_5P_6$	

0.0.2	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_3P_5P_6$	$\beta = (\frac{7}{34}, \frac{7}{34}, \frac{7}{34}, \frac{5}{34}, \frac{5}{34}, \frac{3}{34})$
		$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4P_5P_6$	
		$P_2P_3P_4P_5P_6$	

$0.0.3a$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	$\beta = (\frac{8}{36}, \frac{8}{36}, \frac{8}{36}, \frac{4}{36}, \frac{4}{36}, \frac{4}{36})$
		$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4P_5P_6$	
		$P_2P_3P_4P_5P_6$	

$0.0.3b$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_3P_5P_6$	$\beta = (\frac{8}{36}, \frac{8}{36}, \frac{6}{36}, \frac{6}{36}, \frac{6}{36}, \frac{2}{36})$
		$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4P_5P_6$	
		$P_2P_3P_4P_5P_6$	

$0.0.4a$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	$\beta = (\frac{9}{38}, \frac{9}{38}, \frac{7}{38}, \frac{5}{38}, \frac{5}{38}, \frac{3}{38})$
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4P_5P_6$	
		$P_2P_3P_4P_5P_6$	

$0.0.4b$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_4P_5$ $P_1P_3P_4P_5$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{9}{38}, \frac{7}{38}, \frac{7}{38}, \frac{7}{38}, \frac{7}{38}, \frac{1}{38})$
$0.0.5a$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_6$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{10}{40}, \frac{10}{40}, \frac{6}{40}, \frac{6}{40}, \frac{4}{40}, \frac{4}{40})$
$0.0.5b$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_3P_4P_5$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{10}{40}, \frac{8}{40}, \frac{8}{40}, \frac{6}{40}, \frac{6}{40}, \frac{2}{40})$
$0.0.5c$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{8}{40}, \frac{8}{40}, \frac{8}{40}, \frac{8}{40}, \frac{8}{40}, 0)$
$0.0.6a$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{11}{42}, \frac{9}{42}, \frac{7}{42}, \frac{7}{42}, \frac{5}{42}, \frac{3}{42})$
$0.0.6b$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_6$ $P_1P_2P_5P_6$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{11}{42}, \frac{11}{42}, \frac{5}{42}, \frac{5}{42}, \frac{5}{42}, \frac{5}{42})$
$0.0.6c$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{9}{42}, \frac{9}{42}, \frac{9}{42}, \frac{7}{42}, \frac{7}{42}, \frac{1}{42})$
$0.0.7a$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_4P_5$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$	$\beta = (\frac{12}{44}, \frac{10}{44}, \frac{6}{44}, \frac{6}{44}, \frac{6}{44}, \frac{4}{44})$

$0.0.7b$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	$\beta = (\frac{12}{44}, \frac{8}{44}, \frac{8}{44}, \frac{8}{44}, \frac{4}{44}, \frac{4}{44})$
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_6$		

$0.0.7c$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	$\beta = (\frac{10}{44}, \frac{10}{44}, \frac{8}{44}, \frac{8}{44}, \frac{6}{44}, \frac{2}{44})$
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$	$P_2P_3P_4P_5P_6$	
	$P_2P_3P_4P_5$		

$0.0.8a$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{13}{46}, \frac{9}{46}, \frac{7}{46}, \frac{7}{46}, \frac{5}{46}, \frac{5}{46})$
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		

$0.0.8b$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	$\beta = (\frac{11}{46}, \frac{11}{46}, \frac{7}{46}, \frac{7}{46}, \frac{7}{46}, \frac{3}{46})$
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_2P_3P_4P_5$		

$0.0.8c$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{11}{46}, \frac{9}{46}, \frac{9}{46}, \frac{9}{46}, \frac{5}{46}, \frac{3}{46})$
	$P_1P_3P_4P_5$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_6$		
	$P_2P_3P_4P_5$		

$0.0.9a$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{12}{48}, \frac{10}{48}, \frac{8}{48}, \frac{8}{48}, \frac{6}{48}, \frac{4}{48})$
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		
	$P_2P_3P_4P_5$		

$0.0.9b$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{14}{48}, \frac{8}{48}, \frac{8}{48}, \frac{6}{48}, \frac{6}{48}, \frac{6}{48})$
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		
	$P_1P_3P_5P_6$		

$0.0.9c$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{10}{48}, \frac{10}{48}, \frac{10}{48}, \frac{10}{48}, \frac{4}{48}, \frac{4}{48})$
	$P_1P_3P_4P_5$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_6$		
	$P_2P_3P_4P_5$		
	$P_2P_3P_4P_6$		

$0.0.10a$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{11}{50}, \frac{11}{50}, \frac{9}{50}, \frac{9}{50}, \frac{5}{50}, \frac{5}{50})$
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		
	$P_2P_3P_4P_5$		
	$P_2P_3P_4P_6$		

$0.0.10b$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{13}{50}, \frac{9}{50}, \frac{9}{50}, \frac{7}{50}, \frac{7}{50}, \frac{5}{50})$
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		
	$P_1P_3P_5P_6$		
	$P_2P_3P_4P_5$		

$0.0.10c$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{15}{50}, \frac{7}{50}, \frac{7}{50}, \frac{7}{50}, \frac{7}{50}, \frac{7}{50})$
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		
	$P_1P_3P_5P_6$		
	$P_1P_4P_5P_6$		

$$
\begin{array}{llll}
0.0.11a & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 & \\
 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 & \\
 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 & \\
 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 & \\
 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 & \\
 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 & \beta = (\frac{12}{52}, \frac{10}{52}, \frac{10}{52}, \frac{8}{52}, \frac{6}{52}, \frac{6}{52}) \\
 & P_1P_3P_4P_5 & & \\
 & P_1P_3P_4P_6 & & \\
 & P_1P_3P_5P_6 & & \\
 & P_2P_3P_4P_5 & & \\
 & P_2P_3P_4P_6 & &
\end{array}
$$

$$
\begin{array}{llll}
0.0.11b & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 & \\
 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 & \\
 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 & \\
 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 & \\
 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 & \\
 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 & \beta = (\frac{14}{52}, \frac{8}{52}, \frac{8}{52}, \frac{8}{52}, \frac{8}{52}, \frac{6}{52}) \\
 & P_1P_3P_4P_5 & & \\
 & P_1P_3P_4P_6 & & \\
 & P_1P_3P_5P_6 & & \\
 & P_1P_4P_5P_6 & & \\
 & P_2P_3P_4P_5 & &
\end{array}
$$

$$
\begin{array}{llll}
0.0.12a & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 & \\
 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 & \\
 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 & \\
 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 & \\
 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 & \\
 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 & \\
 & P_1P_3P_4P_5 & & \beta = (\frac{13}{54}, \frac{9}{54}, \frac{9}{54}, \frac{9}{54}, \frac{7}{54}, \frac{7}{54}) \\
 & P_1P_3P_4P_6 & & \\
 & P_1P_3P_5P_6 & & \\
 & P_1P_4P_5P_6 & & \\
 & P_2P_3P_4P_5 & & \\
 & P_2P_3P_4P_6 & &
\end{array}
$$

$$
\begin{array}{llll}
0.0.12b & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 & \\
 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 & \\
 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 & \\
 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 & \\
 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 & \\
 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 & \\
 & P_1P_3P_4P_5 & & \beta = (\frac{11}{54}, \frac{11}{54}, \frac{11}{54}, \frac{7}{54}, \frac{7}{54}, \frac{7}{54}) \\
 & P_1P_3P_4P_6 & & \\
 & P_1P_3P_5P_6 & & \\
 & P_2P_3P_4P_5 & & \\
 & P_2P_3P_4P_6 & & \\
 & P_2P_3P_5P_6 & &
\end{array}
$$

0.0.13	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		$\beta = (\frac{12}{56}, \frac{10}{56}, \frac{10}{56}, \frac{8}{56}, \frac{8}{56}, \frac{8}{56})$
	$P_1P_3P_4P_6$		
	$P_1P_3P_5P_6$		
	$P_1P_4P_5P_6$		
	$P_2P_3P_4P_5$		
	$P_2P_3P_4P_6$		
	$P_2P_3P_5P_6$		

0.0.14	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		$\beta = (\frac{11}{58}, \frac{11}{58}, \frac{9}{58}, \frac{9}{58}, \frac{9}{58}, \frac{9}{58})$
	$P_1P_3P_4P_6$		
	$P_1P_3P_5P_6$		
	$P_1P_4P_5P_6$		
	$P_2P_3P_4P_5$		
	$P_2P_3P_4P_6$		
	$P_2P_3P_5P_6$		
	$P_2P_4P_5P_6$		

0.0.15	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_4P_5$		
	$P_1P_3P_4P_6$		$\beta = (\frac{10}{60}, \frac{10}{60}, \frac{10}{60}, \frac{10}{60}, \frac{10}{60}, \frac{10}{60})$
	$P_1P_3P_5P_6$		
	$P_1P_4P_5P_6$		
	$P_2P_3P_4P_5$		
	$P_2P_3P_4P_6$		
	$P_2P_3P_5P_6$		
	$P_2P_4P_5P_6$		
	$P_3P_4P_5P_6$		

0.1.3	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	$\beta = (\frac{9}{36}, \frac{9}{36}, \frac{9}{36}, \frac{9}{36}, \frac{3}{36}, \frac{3}{36})$
			$P_1P_2P_4P_5P_6$	
			$P_1P_3P_4P_5P_6$	
			$P_2P_3P_4P_5P_6$	

0.1.4	$P_1P_2P_4P_5$	$\beta = (\frac{10}{38}, \frac{10}{38}, \frac{8}{38}, \frac{4}{38}, \frac{4}{38}, \frac{2}{38})$

$0.1.5a$	$P_1P_2P_4P_5$	$\beta = (\frac{11}{40}, \frac{11}{40}, \frac{7}{40}, \frac{5}{40}, \frac{3}{40}, \frac{3}{40})$
	$P_1P_2P_4P_6$	

0.1.5b $P_1P_2P_4P_5$
$P_1P_3P_4P_5$ $\beta = (\frac{11}{40}, \frac{9}{40}, \frac{9}{40}, \frac{5}{40}, \frac{5}{40}, \frac{1}{40})$

0.1.6a $P_1P_2P_4P_5$
$P_1P_2P_4P_6$ $\beta = (\frac{12}{42}, \frac{12}{42}, \frac{6}{42}, \frac{4}{42}, \frac{4}{42}, \frac{4}{42})$
$P_1P_2P_5P_6$

0.1.6b $P_1P_2P_4P_5$
$P_1P_3P_4P_5$ $\beta = (\frac{12}{42}, \frac{10}{42}, \frac{8}{42}, \frac{6}{42}, \frac{4}{42}, \frac{2}{42})$
$P_1P_2P_4P_6$

0.1.6c $P_1P_2P_4P_5$
$P_1P_3P_4P_5$ $\beta = (\frac{10}{42}, \frac{10}{42}, \frac{10}{42}, \frac{6}{42}, \frac{6}{42}, 0)$
$P_2P_3P_4P_5$

0.1.7a $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_2P_5P_6$ $\beta = (\frac{13}{44}, \frac{11}{44}, \frac{7}{44}, \frac{5}{44}, \frac{5}{44}, \frac{3}{44})$

0.1.7b $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $\beta = (\frac{13}{44}, \frac{9}{44}, \frac{9}{44}, \frac{7}{44}, \frac{3}{44}, \frac{3}{44})$

0.1.7c $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_2P_3P_4P_5$ $\beta = (\frac{11}{44}, \frac{11}{44}, \frac{9}{44}, \frac{7}{44}, \frac{5}{44}, \frac{1}{44})$

0.1.8a $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $\beta = (\frac{14}{46}, \frac{10}{46}, \frac{8}{46}, \frac{6}{46}, \frac{4}{46}, \frac{4}{46})$
$P_1P_2P_5P_6$

0.1.8b $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_2P_3P_4P_5$ $\beta = (\frac{12}{46}, \frac{12}{46}, \frac{8}{46}, \frac{6}{46}, \frac{6}{46}, \frac{2}{46})$
$P_1P_2P_5P_6$

0.1.8c $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $\beta = (\frac{12}{46}, \frac{10}{46}, \frac{10}{46}, \frac{8}{46}, \frac{4}{46}, \frac{2}{46})$
$P_2P_3P_4P_5$

0.1.9a $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $\beta = (\frac{15}{48}, \frac{9}{48}, \frac{9}{48}, \frac{5}{48}, \frac{5}{48}, \frac{5}{48})$
$P_1P_2P_5P_6$ $P_1P_3P_5P_6$

0.1.9b $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $\beta = (\frac{13}{48}, \frac{11}{48}, \frac{9}{48}, \frac{7}{48}, \frac{5}{48}, \frac{3}{48})$
$P_1P_2P_5P_6$ $P_2P_3P_4P_5$

0.1.9c $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $\beta = (\frac{11}{48}, \frac{11}{48}, \frac{11}{48}, \frac{9}{48}, \frac{3}{48}, \frac{3}{48})$
$P_2P_3P_4P_5$ $P_2P_3P_4P_6$

0.1.10a $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$
$P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $\beta = (\frac{16}{50}, \frac{8}{50}, \frac{8}{50}, \frac{6}{50}, \frac{6}{50}, \frac{6}{50})$
$P_1P_4P_5P_6$

0.1.10b $P_1P_2P_4P_5$ $P_1P_3P_4P_5$
$P_1P_2P_4P_6$ $P_1P_3P_4P_6$
$P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $\beta = (\frac{14}{50}, \frac{10}{50}, \frac{10}{50}, \frac{6}{50}, \frac{6}{50}, \frac{4}{50})$
$P_2P_3P_4P_5$

0.1.10c $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_1P_2P_5P_6$ $P_2P_3P_4P_5$ $P_2P_3P_4P_6$ $\beta = (\frac{12}{50}, \frac{12}{50}, \frac{10}{50}, \frac{8}{50}, \frac{4}{50}, \frac{4}{50})$

0.1.11a $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_1P_4P_5P_6$ $P_2P_3P_4P_5$ $\beta = (\frac{15}{52}, \frac{9}{52}, \frac{9}{52}, \frac{7}{52}, \frac{7}{52}, \frac{5}{52})$

0.1.11b $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_2P_3P_4P_5$ $P_2P_3P_4P_6$ $\beta = (\frac{13}{52}, \frac{11}{52}, \frac{11}{52}, \frac{7}{52}, \frac{5}{52}, \frac{5}{52})$

0.1.12a $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_1P_4P_5P_6$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_2P_3P_4P_5$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_2P_3P_4P_6$ $\beta = (\frac{14}{54}, \frac{10}{54}, \frac{10}{54}, \frac{8}{54}, \frac{6}{54}, \frac{6}{54})$

0.1.12b $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_2P_3P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_2P_3P_5P_6$ $\beta = (\frac{12}{54}, \frac{12}{54}, \frac{12}{54}, \frac{6}{54}, \frac{6}{54}, \frac{6}{54})$

0.1.13 $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_2P_3P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_2P_3P_5P_6$ $P_1P_4P_5P_6$ $\beta = (\frac{13}{56}, \frac{11}{56}, \frac{11}{56}, \frac{7}{56}, \frac{7}{56}, \frac{7}{56})$

0.1.14 $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_2P_3P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_2P_3P_5P_6$ $P_1P_4P_5P_6$ $P_2P_4P_5P_6$ $\beta = (\frac{12}{58}, \frac{12}{58}, \frac{10}{58}, \frac{8}{58}, \frac{8}{58}, \frac{8}{58})$

0.1.15 $P_1P_2P_4P_5$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_6$ $P_2P_3P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_5P_6$ $P_2P_3P_5P_6$ $P_1P_4P_5P_6$ $P_2P_4P_5P_6$ $P_3P_4P_5P_6$ $\beta = (\frac{11}{60}, \frac{11}{60}, \frac{11}{60}, \frac{9}{60}, \frac{9}{60}, \frac{9}{60})$

0.2.5 $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$ $P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$ $\beta = (\frac{12}{40}, \frac{12}{40}, \frac{6}{40}, \frac{6}{40}, \frac{2}{40}, \frac{2}{40})$

0.2.6a $P_1P_3P_4P_5$ $\beta = (\frac{13}{42}, \frac{11}{42}, \frac{7}{42}, \frac{7}{42}, \frac{3}{42}, \frac{1}{42})$

0.2.6b $P_1P_2P_5P_6$ $\beta = (\frac{13}{42}, \frac{13}{42}, \frac{5}{42}, \frac{5}{42}, \frac{3}{42}, \frac{3}{42})$

0.2.7a $P_1P_3P_4P_6$ $P_1P_3P_5P_6$ $\beta = (\frac{14}{44}, \frac{10}{44}, \frac{8}{44}, \frac{8}{44}, \frac{2}{44}, \frac{2}{44})$

0.2.7b $P_1P_3P_4P_5$ $P_2P_3P_4P_5$ $\beta = (\frac{12}{44}, \frac{12}{44}, \frac{8}{44}, \frac{8}{44}, \frac{4}{44}, 0)$

0.2.7c $P_1P_2P_5P_6$ $P_1P_3P_4P_5$ $\beta = (\frac{14}{44}, \frac{12}{44}, \frac{6}{44}, \frac{6}{44}, \frac{4}{44}, \frac{2}{44})$

$$0.2.8a \quad \begin{array}{l} P_1P_3P_4P_6 \\ P_2P_3P_4P_5 \\ P_1P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{13}{46}, \tfrac{11}{49}, \tfrac{9}{46}, \tfrac{9}{46}, \tfrac{3}{46}, \tfrac{1}{46})$$

$$0.2.8b \quad \begin{array}{l} P_1P_2P_5P_6 \\ P_1P_3P_4P_5 \\ P_1P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{15}{46}, \tfrac{11}{46}, \tfrac{7}{46}, \tfrac{7}{46}, \tfrac{3}{46}, \tfrac{3}{46})$$

$$0.2.8c \quad \begin{array}{l} P_1P_2P_5P_6 \\ P_1P_3P_4P_5 \\ P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{13}{46}, \tfrac{13}{46}, \tfrac{7}{46}, \tfrac{7}{46}, \tfrac{5}{46}, \tfrac{1}{46})$$

$$0.2.9a \quad \begin{array}{ll} P_1P_3P_4P_6 & P_2P_3P_4P_5 \\ P_1P_3P_5P_6 & P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{12}{48}, \tfrac{12}{48}, \tfrac{10}{48}, \tfrac{10}{48}, \tfrac{2}{48}, \tfrac{2}{48})$$

$$0.2.9b \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_3P_4P_5 \\ P_1P_3P_4P_6 & P_1P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{48}, \tfrac{10}{48}, \tfrac{8}{48}, \tfrac{6}{48}, \tfrac{4}{48}, \tfrac{4}{48})$$

$$0.2.9c \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_3P_4P_6 \\ P_1P_3P_4P_5 & P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{14}{48}, \tfrac{12}{48}, \tfrac{8}{48}, \tfrac{8}{48}, \tfrac{4}{48}, \tfrac{2}{48})$$

$$0.2.10a \quad \begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_1P_3P_4P_5 & P_2P_3P_4P_6 \\ P_1P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{13}{50}, \tfrac{13}{50}, \tfrac{9}{50}, \tfrac{9}{50}, \tfrac{3}{50}, \tfrac{3}{50})$$

$$0.2.10b \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_3P_5P_6 \\ P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ P_1P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{15}{50}, \tfrac{11}{50}, \tfrac{9}{50}, \tfrac{7}{50}, \tfrac{5}{50}, \tfrac{3}{50})$$

$$0.2.10c \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_3P_5P_6 \\ P_1P_3P_4P_5 & P_1P_4P_5P_6 \\ P_1P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{17}{50}, \tfrac{9}{50}, \tfrac{7}{50}, \tfrac{7}{50}, \tfrac{5}{50}, \tfrac{5}{50})$$

$$0.2.11a \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_3P_5P_6 \\ P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ P_1P_3P_4P_6 & P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{14}{52}, \tfrac{12}{52}, \tfrac{10}{52}, \tfrac{8}{52}, \tfrac{4}{52}, \tfrac{4}{52})$$

$$0.2.11b \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_3P_5P_6 \\ P_1P_3P_4P_5 & P_1P_4P_5P_6 \\ P_1P_3P_4P_6 & P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{16}{52}, \tfrac{10}{52}, \tfrac{8}{52}, \tfrac{8}{52}, \tfrac{6}{52}, \tfrac{4}{52})$$

$$0.2.12a \quad \begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_1P_3P_4P_5 & P_2P_3P_4P_6 \\ P_1P_3P_4P_6 & P_2P_3P_5P_6 \\ P_1P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{13}{54}, \tfrac{13}{54}, \tfrac{11}{54}, \tfrac{7}{54}, \tfrac{5}{54}, \tfrac{5}{54})$$

$$0.2.12b \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_4P_5P_6 \\ P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ P_1P_3P_4P_6 & P_2P_3P_4P_6 \\ P_1P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{15}{54}, \tfrac{11}{54}, \tfrac{9}{54}, \tfrac{9}{54}, \tfrac{5}{54}, \tfrac{5}{54})$$

$$0.2.13 \quad \begin{array}{ll} P_1P_2P_5P_6 & P_1P_4P_5P_6 \\ P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ P_1P_3P_4P_6 & P_2P_3P_4P_6 \\ P_1P_3P_5P_6 & P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{14}{56}, \tfrac{12}{56}, \tfrac{10}{56}, \tfrac{8}{56}, \tfrac{6}{56}, \tfrac{6}{56})$$

0.2.14

$P_1P_2P_5P_6$ $P_2P_3P_4P_5$
$P_1P_3P_4P_5$ $P_2P_3P_4P_6$
$P_1P_3P_4P_6$ $P_2P_3P_5P_6$
$P_1P_3P_5P_6$ $P_2P_4P_5P_6$
$P_1P_4P_5P_6$

$\beta = (\frac{13}{58}, \frac{13}{58}, \frac{9}{58}, \frac{9}{58}, \frac{7}{58}, \frac{7}{58})$

0.2.15

$P_1P_2P_5P_6$ $P_2P_3P_4P_5$
$P_1P_3P_4P_5$ $P_2P_3P_4P_6$
$P_1P_3P_4P_6$ $P_2P_3P_5P_6$
$P_1P_3P_5P_6$ $P_2P_4P_5P_6$
$P_1P_4P_5P_6$ $P_3P_4P_5P_6$

$\beta = (\frac{12}{60}, \frac{12}{60}, \frac{10}{60}, \frac{10}{60}, \frac{8}{60}, \frac{8}{60})$

$0.3a.6$

$P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$

$\beta = (\frac{14}{42}, \frac{14}{42}, \frac{4}{42}, \frac{4}{42}, \frac{4}{42}, \frac{2}{42})$

$0.3a.7$ $P_1P_3P_4P_5$ $\beta = (\frac{15}{44}, \frac{13}{44}, \frac{5}{44}, \frac{5}{44}, \frac{5}{44}, \frac{1}{44})$

$0.3a.8a$

$P_1P_3P_4P_5$
$P_1P_3P_4P_6$

$\beta = (\frac{16}{46}, \frac{12}{46}, \frac{6}{46}, \frac{6}{46}, \frac{4}{46}, \frac{2}{46})$

$0.3a.8b$

$P_1P_3P_4P_5$
$P_2P_3P_4P_5$

$\beta = (\frac{14}{46}, \frac{14}{46}, \frac{6}{46}, \frac{6}{46}, \frac{6}{46}, 0)$

$0.3a.8c$

$P_1P_3P_4P_5$
$P_1P_3P_4P_6$

$\beta = (\frac{14}{46}, \frac{10}{46}, \frac{10}{46}, \frac{10}{46}, \frac{2}{46}, 0)$

$0.3a.9a$

$P_1P_3P_4P_5$
$P_2P_3P_4P_5$
$P_1P_3P_4P_6$

$\beta = (\frac{15}{48}, \frac{13}{48}, \frac{7}{48}, \frac{7}{48}, \frac{5}{48}, \frac{1}{48})$

$0.3a.9b$

$P_1P_3P_4P_5$
$P_1P_3P_4P_6$
$P_1P_3P_5P_6$

$\beta = (\frac{17}{48}, \frac{11}{48}, \frac{7}{48}, \frac{5}{48}, \frac{5}{48}, \frac{3}{48})$

$0.3a.10a$

$P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$

$\beta = (\frac{14}{50}, \frac{14}{50}, \frac{8}{50}, \frac{8}{50}, \frac{4}{50}, \frac{2}{50})$

$0.3a.10b$

$P_1P_3P_4P_5$ $P_1P_3P_5P_6$
$P_1P_3P_4P_6$ $P_2P_3P_4P_5$

$\beta = (\frac{16}{50}, \frac{12}{50}, \frac{8}{50}, \frac{6}{50}, \frac{6}{50}, \frac{2}{50})$

$0.3a.10c$

$P_1P_3P_4P_5$ $P_1P_3P_5P_6$
$P_1P_3P_4P_6$ $P_1P_4P_5P_6$

$\beta = (\frac{18}{50}, \frac{10}{50}, \frac{6}{50}, \frac{6}{50}, \frac{6}{50}, \frac{4}{50})$

$0.3a.11a$

$P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$
$P_1P_3P_5P_6$

$\beta = (\frac{15}{52}, \frac{13}{52}, \frac{9}{52}, \frac{7}{52}, \frac{5}{52}, \frac{3}{52})$

$0.3a.11b$

$P_1P_3P_4P_5$ $P_1P_4P_5P_6$
$P_1P_3P_4P_6$ $P_2P_3P_4P_5$
$P_1P_3P_5P_6$

$\beta = (\frac{17}{52}, \frac{11}{52}, \frac{7}{52}, \frac{7}{52}, \frac{7}{52}, \frac{3}{52})$

$0.3a.12a$

$P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$

$\beta = (\frac{14}{54}, \frac{14}{54}, \frac{10}{54}, \frac{6}{54}, \frac{6}{54}, \frac{4}{54})$

$0.3a.12b$ $P_1P_3P_4P_5$ $P_1P_4P_5P_6$
$P_1P_3P_4P_6$ $P_2P_3P_4P_5$
$P_1P_3P_5P_6$ $P_2P_3P_4P_6$
$\beta = (\frac{16}{54}, \frac{12}{54}, \frac{8}{54}, \frac{8}{54}, \frac{6}{54}, \frac{4}{54})$

$0.3a.13$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$
$P_1P_4P_5P_6$
$\beta = (\frac{15}{56}, \frac{13}{56}, \frac{9}{56}, \frac{7}{56}, \frac{7}{56}, \frac{5}{56})$

$0.3a.14$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$
$P_1P_4P_5P_6$ $P_2P_4P_5P_6$
$\beta = (\frac{14}{58}, \frac{14}{58}, \frac{8}{58}, \frac{8}{58}, \frac{8}{58}, \frac{6}{58})$

$0.3a.15$ $P_1P_3P_4P_5$ $P_2P_3P_4P_6$
$P_1P_3P_4P_6$ $P_2P_3P_5P_6$
$P_1P_3P_5P_6$ $P_2P_4P_5P_6$
$P_1P_4P_5P_6$ $P_3P_4P_5P_6$
$P_2P_3P_4P_5$
$\beta = (\frac{13}{60}, \frac{13}{60}, \frac{9}{60}, \frac{9}{60}, \frac{9}{60}, \frac{7}{60})$

$0.3b.7$ $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_3P_4$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_1P_3P_4P_5$ $P_2P_3P_4P_5P_6$
$P_1P_3P_4P_6$
$\beta = (\frac{15}{44}, \frac{9}{44}, \frac{9}{44}, \frac{9}{44}, \frac{1}{44}, \frac{1}{44})$

$0.3b.8$ $P_1P_2P_5P_6$ $\beta = (\frac{16}{46}, \frac{10}{46}, \frac{8}{46}, \frac{8}{46}, \frac{2}{46}, \frac{2}{46})$

$0.3b.9a$ $P_1P_2P_5P_6$
$P_1P_3P_5P_6$
$\beta = (\frac{17}{48}, \frac{9}{48}, \frac{9}{48}, \frac{7}{48}, \frac{3}{48}, \frac{3}{48})$

$0.3b.9b$ $P_1P_2P_5P_6$
$P_2P_3P_4P_5$
$\beta = (\frac{15}{48}, \frac{11}{48}, \frac{9}{48}, \frac{9}{48}, \frac{3}{48}, \frac{1}{48})$

$0.3b.9c$ $P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$\beta = (\frac{13}{48}, \frac{11}{48}, \frac{11}{48}, \frac{11}{48}, \frac{1}{48}, \frac{1}{48})$

$0.3b.10a$ $P_1P_2P_5P_6$
$P_1P_3P_5P_6$
$P_1P_4P_5P_6$
$\beta = (\frac{18}{50}, \frac{8}{50}, \frac{8}{50}, \frac{8}{50}, \frac{4}{50}, \frac{4}{50})$

$0.3b.10b$ $P_1P_2P_5P_6$
$P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$\beta = (\frac{14}{50}, \frac{12}{50}, \frac{10}{50}, \frac{10}{50}, \frac{2}{50}, \frac{2}{50})$

$0.3b.10c$ $P_1P_2P_5P_6$
$P_2P_3P_4P_5$
$P_1P_3P_5P_6$
$\beta = (\frac{16}{50}, \frac{10}{50}, \frac{10}{50}, \frac{8}{50}, \frac{4}{50}, \frac{2}{50})$

$0.3b.11a$ $P_1P_2P_5P_6$
$P_1P_3P_5P_6$
$P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$\beta = (\frac{15}{52}, \frac{11}{52}, \frac{11}{52}, \frac{9}{52}, \frac{3}{52}, \frac{3}{52})$

$0.3b.11b$ $P_1P_2P_5P_6$
$P_2P_3P_4P_5$
$P_1P_3P_5P_6$
$P_1P_4P_5P_6$
$\beta = (\frac{17}{52}, \frac{9}{52}, \frac{9}{52}, \frac{9}{52}, \frac{5}{52}, \frac{3}{52})$

$0.3b.12a$
$$\begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_1P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{16}{54}, \tfrac{10}{54}, \tfrac{10}{54}, \tfrac{10}{54}, \tfrac{4}{54}, \tfrac{4}{54})$$

$0.3b.12b$
$$\begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_2P_3P_4P_5 & P_2P_3P_5P_6 \\ P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{16}{54}, \tfrac{12}{54}, \tfrac{8}{54}, \tfrac{8}{54}, \tfrac{6}{54}, \tfrac{4}{54})$$

$0.3b.13$
$$\begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_1P_4P_5P_6 & P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{15}{56}, \tfrac{11}{56}, \tfrac{11}{56}, \tfrac{9}{56}, \tfrac{5}{56}, \tfrac{5}{56})$$

$0.3b.14$
$$\begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_1P_4P_5P_6 & P_2P_3P_5P_6 \\ P_2P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{14}{58}, \tfrac{12}{58}, \tfrac{10}{58}, \tfrac{10}{58}, \tfrac{6}{58}, \tfrac{6}{58})$$

$0.3b.15$
$$\begin{array}{ll} P_1P_2P_5P_6 & P_2P_3P_4P_5 \\ P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_1P_4P_5P_6 & P_2P_3P_5P_6 \\ P_2P_4P_5P_6 & P_3P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{13}{60}, \tfrac{11}{60}, \tfrac{11}{60}, \tfrac{11}{60}, \tfrac{7}{60}, \tfrac{7}{60})$$

$0.4a.6$
$$\begin{array}{lll} P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{15}{42}, \tfrac{15}{42}, \tfrac{3}{42}, \tfrac{3}{42}, \tfrac{3}{42}, \tfrac{3}{42})$$

$0.4a.7$
$$P_1P_3P_4P_5 \qquad \beta = (\tfrac{16}{44}, \tfrac{14}{44}, \tfrac{4}{44}, \tfrac{4}{44}, \tfrac{4}{44}, \tfrac{2}{44})$$

$0.4a.8a$
$$\begin{array}{l} P_1P_3P_4P_5 \\ P_1P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{17}{46}, \tfrac{13}{46}, \tfrac{5}{46}, \tfrac{5}{46}, \tfrac{3}{46}, \tfrac{3}{46})$$

$0.4a.8b$
$$\begin{array}{l} P_1P_3P_4P_5 \\ P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{15}{46}, \tfrac{15}{46}, \tfrac{5}{46}, \tfrac{5}{46}, \tfrac{5}{46}, \tfrac{1}{46})$$

$0.4a.9a$
$$\begin{array}{l} P_1P_3P_4P_5 \\ P_1P_3P_4P_6 \\ P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{16}{48}, \tfrac{14}{48}, \tfrac{6}{48}, \tfrac{6}{48}, \tfrac{4}{48}, \tfrac{2}{48})$$

$0.4a.9b$
$$\begin{array}{l} P_1P_3P_4P_5 \\ P_1P_3P_4P_6 \\ P_1P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{18}{48}, \tfrac{12}{48}, \tfrac{6}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{4}{48})$$

$0.4a.10a$
$$\begin{array}{ll} P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ P_1P_3P_4P_6 & P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{15}{50}, \tfrac{15}{50}, \tfrac{7}{50}, \tfrac{7}{50}, \tfrac{3}{50}, \tfrac{3}{50})$$

$0.4a.10b$
$$\begin{array}{ll} P_1P_3P_4P_5 & P_1P_3P_5P_6 \\ P_1P_3P_4P_6 & P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{17}{50}, \tfrac{13}{50}, \tfrac{7}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{3}{50})$$

$0.4a.10c$
$$\begin{array}{ll} P_1P_3P_4P_5 & P_1P_3P_5P_6 \\ P_1P_3P_4P_6 & P_1P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{19}{50}, \tfrac{11}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{5}{50})$$

$0.4a.11a$
$$\begin{array}{ll} P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ P_1P_3P_4P_6 & P_2P_3P_4P_6 \\ P_1P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{16}{52}, \tfrac{14}{52}, \tfrac{8}{52}, \tfrac{6}{52}, \tfrac{4}{52}, \tfrac{4}{52})$$

$0.4a.11b$ $P_1P_3P_4P_5$ $P_1P_4P_5P_6$
$P_1P_3P_4P_6$ $P_2P_3P_4P_5$ $\beta = (\frac{18}{52}, \frac{12}{52}, \frac{6}{52}, \frac{6}{52}, \frac{6}{52}, \frac{4}{52})$
$P_1P_3P_5P_6$

$0.4a.12a$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$ $\beta = (\frac{15}{54}, \frac{15}{54}, \frac{9}{54}, \frac{5}{54}, \frac{5}{54}, \frac{5}{54})$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$

$0.4a.12b$ $P_1P_3P_4P_5$ $P_1P_4P_5P_6$
$P_1P_3P_4P_6$ $P_2P_3P_4P_5$ $\beta = (\frac{17}{54}, \frac{13}{54}, \frac{7}{54}, \frac{7}{54}, \frac{5}{54}, \frac{5}{54})$
$P_1P_3P_5P_6$ $P_2P_3P_4P_6$

$0.4a.13$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$ $\beta = (\frac{16}{56}, \frac{14}{56}, \frac{8}{56}, \frac{6}{56}, \frac{6}{56}, \frac{6}{56})$
$P_1P_4P_5P_6$

$0.4a.14$ $P_1P_3P_4P_5$ $P_2P_3P_4P_5$
$P_1P_3P_4P_6$ $P_2P_3P_4P_6$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$ $\beta = (\frac{15}{58}, \frac{15}{58}, \frac{7}{58}, \frac{7}{58}, \frac{7}{58}, \frac{7}{58})$
$P_1P_4P_5P_6$ $P_2P_4P_5P_6$

$0.4a.15$ $P_1P_3P_4P_5$ $P_2P_3P_4P_6$
$P_1P_3P_4P_6$ $P_2P_3P_5P_6$
$P_1P_3P_5P_6$ $P_2P_4P_5P_6$ $\beta = (\frac{14}{60}, \frac{14}{60}, \frac{8}{60}, \frac{8}{60}, \frac{8}{60}, \frac{8}{60})$
$P_1P_4P_5P_6$ $P_3P_4P_5P_6$
$P_2P_3P_4P_5$

$0.4b.8$ $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_3P_4$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$ $\beta = (\frac{17}{46}, \frac{11}{46}, \frac{7}{46}, \frac{7}{46}, \frac{3}{46}, \frac{1}{46})$
$P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$
$P_1P_3P_4P_5$
$P_1P_3P_4P_6$

$0.4b.9a$ $P_1P_3P_5P_6$ $\beta = (\frac{18}{48}, \frac{10}{48}, \frac{8}{48}, \frac{6}{48}, \frac{4}{48}, \frac{2}{48})$

$0.4b.9b$ $P_2P_3P_4P_5$ $\beta = (\frac{16}{48}, \frac{12}{48}, \frac{8}{48}, \frac{8}{48}, \frac{4}{48}, 0)$

$0.4b.10a$ $P_1P_3P_5P_6$ $\beta = (\frac{19}{50}, \frac{9}{50}, \frac{7}{50}, \frac{7}{50}, \frac{5}{50}, \frac{3}{50})$
$P_1P_4P_5P_6$

$0.4b.10b$ $P_2P_3P_4P_5$ $\beta = (\frac{15}{50}, \frac{13}{50}, \frac{9}{50}, \frac{9}{50}, \frac{3}{50}, \frac{1}{50})$
$P_2P_3P_4P_6$

$0.4b.10c$ $P_1P_3P_5P_6$ $\beta = (\frac{17}{50}, \frac{11}{50}, \frac{9}{50}, \frac{7}{50}, \frac{5}{50}, \frac{1}{50})$
$P_2P_3P_4P_5$

$0.4b.11a$ $P_1P_3P_5P_6$
$P_2P_3P_4P_5$ $\beta = (\frac{18}{52}, \frac{10}{52}, \frac{8}{52}, \frac{8}{52}, \frac{6}{52}, \frac{2}{52})$
$P_1P_4P_5P_6$

$0.4b.11b$ $P_1P_3P_5P_6$
$P_2P_3P_4P_5$ $\beta = (\frac{16}{52}, \frac{12}{52}, \frac{10}{52}, \frac{8}{52}, \frac{4}{52}, \frac{2}{52})$
$P_2P_3P_4P_6$

$0.4b.12a$ $P_1P_3P_5P_6$ $P_2P_3P_4P_5$ $\beta = (\frac{17}{54}, \frac{11}{54}, \frac{9}{54}, \frac{9}{54}, \frac{5}{54}, \frac{3}{54})$
$P_1P_4P_5P_6$ $P_2P_3P_4P_6$

$0.4b.12b$ $P_1P_3P_5P_6$ $P_2P_3P_4P_6$
$P_2P_3P_4P_5$ $P_2P_3P_5P_6$
$\beta = (\frac{15}{54}, \frac{13}{54}, \frac{11}{54}, \frac{7}{54}, \frac{5}{54}, \frac{3}{54})$

$0.4b.13$ $P_1P_3P_5P_6$ $P_2P_3P_4P_6$
$P_1P_4P_5P_6$ $P_2P_3P_5P_6$
$P_2P_3P_4P_5$
$\beta = (\frac{16}{56}, \frac{12}{56}, \frac{10}{56}, \frac{8}{56}, \frac{6}{56}, \frac{4}{56})$

$0.4b.14$ $P_1P_3P_5P_6$ $P_2P_3P_4P_6$
$P_1P_4P_5P_6$ $P_2P_3P_5P_6$
$P_2P_3P_4P_5$ $P_2P_4P_5P_6$
$\beta = (\frac{15}{58}, \frac{13}{58}, \frac{9}{58}, \frac{9}{58}, \frac{7}{58}, \frac{5}{58})$

$0.4b.15$ $P_1P_3P_5P_6$ $P_2P_3P_4P_6$
$P_1P_4P_5P_6$ $P_2P_4P_5P_6$
$P_2P_3P_5P_6$ $P_3P_4P_5P_6$
$P_2P_3P_4P_5$
$\beta = (\frac{14}{60}, \frac{12}{60}, \frac{10}{60}, \frac{10}{60}, \frac{8}{60}, \frac{6}{60})$

$0.4c.9$ $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_3P_4$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_2P_3P_4$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_1P_3P_4P_5$ $P_2P_3P_4P_5P_6$
$P_1P_3P_4P_6$
$P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$\beta = (\frac{12}{48}, \frac{12}{48}, \frac{12}{48}, \frac{12}{48}, 0, 0)$

$0.4c.10$ $P_1P_2P_5P_6$
$\beta = (\frac{13}{50}, \frac{13}{50}, \frac{11}{50}, \frac{11}{50}, \frac{1}{50}, \frac{1}{50})$

$0.4c.11$ $P_1P_2P_5P_6$
$P_1P_3P_5P_6$
$\beta = (\frac{14}{52}, \frac{12}{52}, \frac{12}{52}, \frac{10}{52}, \frac{2}{52}, \frac{2}{52})$

$0.4c.12a$ $P_1P_2P_5P_6$
$P_1P_3P_5P_6$
$P_1P_4P_5P_6$
$\beta = (\frac{15}{54}, \frac{11}{54}, \frac{11}{54}, \frac{11}{54}, \frac{3}{54}, \frac{3}{54})$

$0.4c.12b$ $P_1P_2P_5P_6$
$P_1P_3P_5P_6$
$P_2P_3P_5P_6$
$\beta = (\frac{13}{54}, \frac{13}{54}, \frac{13}{54}, \frac{9}{54}, \frac{3}{54}, \frac{3}{54})$

$0.4c.13$ $P_1P_2P_5P_6$ $P_1P_4P_5P_6$
$P_1P_3P_5P_6$ $P_2P_3P_5P_6$
$\beta = (\frac{14}{56}, \frac{12}{56}, \frac{12}{56}, \frac{10}{56}, \frac{4}{56}, \frac{4}{56})$

$0.4c.14$ $P_1P_2P_5P_6$ $P_2P_3P_5P_6$
$P_1P_3P_5P_6$ $P_2P_4P_5P_6$
$P_1P_4P_5P_6$
$\beta = (\frac{13}{58}, \frac{13}{58}, \frac{11}{58}, \frac{11}{58}, \frac{5}{58}, \frac{5}{58})$

$0.4c.15$ $P_1P_2P_5P_6$ $P_2P_3P_5P_6$
$P_1P_3P_5P_6$ $P_2P_4P_5P_6$
$P_1P_4P_5P_6$ $P_3P_4P_5P_6$
$\beta = (\frac{12}{60}, \frac{12}{60}, \frac{12}{60}, \frac{12}{60}, \frac{6}{60}, \frac{6}{60})$

$0.5a.8$ $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_2P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_3P_4$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$
$P_1P_3P_4P_5$
$P_1P_3P_4P_6$
$\beta = (\frac{18}{46}, \frac{12}{46}, \frac{6}{46}, \frac{6}{46}, \frac{2}{46}, \frac{2}{46})$

$0.5a.9a$ $P_1P_3P_5P_6$
$\beta = (\frac{19}{48}, \frac{11}{48}, \frac{7}{48}, \frac{5}{48}, \frac{3}{48}, \frac{3}{48})$

$$0.5a.9b \quad P_2P_3P_4P_5 \qquad \beta = (\tfrac{17}{48}, \tfrac{13}{48}, \tfrac{7}{48}, \tfrac{7}{48}, \tfrac{3}{48}, \tfrac{1}{48})$$

$$0.5a.10a \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_1P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{20}{50}, \tfrac{10}{50}, \tfrac{6}{50}, \tfrac{6}{50}, \tfrac{4}{50}, \tfrac{4}{50})$$

$$0.5a.10b \quad \begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{16}{50}, \tfrac{14}{50}, \tfrac{8}{50}, \tfrac{8}{50}, \tfrac{2}{50}, \tfrac{2}{50})$$

$$0.5a.10c \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{18}{50}, \tfrac{12}{50}, \tfrac{8}{50}, \tfrac{6}{50}, \tfrac{4}{50}, \tfrac{2}{50})$$

$$0.5a.11a \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{19}{52}, \tfrac{11}{52}, \tfrac{7}{52}, \tfrac{7}{52}, \tfrac{5}{52}, \tfrac{3}{52})$$

$$0.5a.11b \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{17}{52}, \tfrac{13}{52}, \tfrac{9}{52}, \tfrac{7}{52}, \tfrac{3}{52}, \tfrac{3}{52})$$

$$0.5a.12a \quad \begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_2P_3P_4P_5 & P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{54}, \tfrac{14}{54}, \tfrac{10}{54}, \tfrac{6}{54}, \tfrac{4}{54}, \tfrac{4}{54})$$

$$0.5a.12b \quad \begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_4P_5 \\ P_1P_4P_5P_6 & P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{18}{54}, \tfrac{12}{54}, \tfrac{8}{54}, \tfrac{8}{54}, \tfrac{4}{54}, \tfrac{4}{54})$$

$$0.5a.13 \quad \begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_1P_4P_5P_6 & P_2P_3P_5P_6 \\ P_2P_3P_4P_5 & \end{array} \qquad \beta = (\tfrac{17}{56}, \tfrac{13}{56}, \tfrac{9}{56}, \tfrac{7}{56}, \tfrac{5}{56}, \tfrac{5}{56})$$

$$0.5a.14 \quad \begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_4P_6 \\ P_1P_4P_5P_6 & P_2P_3P_5P_6 \\ P_2P_3P_4P_5 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{58}, \tfrac{14}{58}, \tfrac{8}{58}, \tfrac{8}{58}, \tfrac{6}{58}, \tfrac{6}{58})$$

$$0.5a.15 \quad \begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_5P_6 \\ P_1P_4P_5P_6 & P_2P_4P_5P_6 \\ P_2P_3P_4P_5 & P_3P_4P_5P_6 \\ P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{15}{60}, \tfrac{13}{60}, \tfrac{9}{60}, \tfrac{9}{60}, \tfrac{7}{60}, \tfrac{7}{60})$$

$$0.5b.10 \quad \begin{array}{lll} P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ P_1P_3P_4 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ P_2P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_4P_5 & \\ & P_1P_3P_4P_6 & \\ & P_2P_3P_4P_5 & \\ & P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{14}{50}, \tfrac{14}{50}, \tfrac{10}{50}, \tfrac{10}{50}, \tfrac{2}{50}, 0)$$

$$0.5b.11 \quad P_1P_3P_5P_6 \qquad \beta = (\tfrac{15}{52}, \tfrac{13}{52}, \tfrac{11}{52}, \tfrac{9}{52}, \tfrac{3}{52}, \tfrac{1}{52})$$

$$0.5b.12a \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_1P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{54}, \tfrac{12}{54}, \tfrac{10}{54}, \tfrac{10}{54}, \tfrac{4}{54}, \tfrac{2}{54})$$

$$0.5b.12b \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{14}{54}, \tfrac{14}{54}, \tfrac{12}{54}, \tfrac{8}{54}, \tfrac{4}{54}, \tfrac{2}{54})$$

$$0.5b.13 \quad \begin{array}{l} P_1P_3P_5P_6 \\ P_1P_4P_5P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{15}{56}, \tfrac{13}{56}, \tfrac{11}{56}, \tfrac{9}{56}, \tfrac{5}{56}, \tfrac{3}{56})$$

$0.5b.14$ $P_1P_3P_5P_6$ $P_2P_3P_5P_6$
$P_1P_4P_5P_6$ $P_2P_4P_5P_6$ $\beta = (\frac{14}{58}, \frac{14}{58}, \frac{10}{58}, \frac{10}{58}, \frac{6}{58}, \frac{4}{58})$

$0.5b.15$ $P_1P_3P_5P_6$ $P_2P_4P_5P_6$
$P_1P_4P_5P_6$ $P_3P_4P_5P_6$ $\beta = (\frac{13}{60}, \frac{13}{60}, \frac{11}{60}, \frac{11}{60}, \frac{7}{60}, \frac{5}{60})$
$P_2P_3P_5P_6$

$0.5c.9$ $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_3P_4$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_3P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$ $\beta = (\frac{19}{48}, \frac{9}{48}, \frac{9}{48}, \frac{5}{48}, \frac{5}{48}, \frac{1}{48})$
$P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$
$P_1P_3P_4P_5$
$P_1P_3P_4P_6$
$P_1P_3P_5P_6$

$0.5c.10a$ $P_1P_4P_5P_6$ $\beta = (\frac{20}{50}, \frac{8}{50}, \frac{8}{50}, \frac{6}{50}, \frac{6}{50}, \frac{2}{50})$

$0.5c.10b$ $P_2P_3P_4P_5$ $\beta = (\frac{18}{50}, \frac{10}{50}, \frac{10}{50}, \frac{6}{50}, \frac{6}{50}, 0)$

$0.5c.11a$ $P_1P_4P_5P_6$
$P_2P_3P_4P_5$ $\beta = (\frac{19}{52}, \frac{9}{52}, \frac{9}{52}, \frac{7}{52}, \frac{7}{52}, \frac{1}{52})$

$0.5c.11b$ $P_2P_3P_4P_5$
$P_2P_3P_4P_6$ $\beta = (\frac{17}{52}, \frac{11}{52}, \frac{11}{52}, \frac{7}{52}, \frac{5}{52}, \frac{1}{52})$

$0.5c.12a$ $P_1P_4P_5P_6$
$P_2P_3P_4P_5$ $\beta = (\frac{18}{54}, \frac{10}{54}, \frac{10}{54}, \frac{8}{54}, \frac{6}{54}, \frac{2}{54})$
$P_2P_3P_4P_6$

$0.5c.12b$ $P_2P_3P_4P_5$
$P_2P_3P_4P_6$ $\beta = (\frac{16}{54}, \frac{12}{54}, \frac{12}{54}, \frac{6}{54}, \frac{6}{54}, \frac{2}{54})$
$P_2P_3P_5P_6$

$0.5c.13$ $P_1P_4P_5P_6$ $P_2P_3P_4P_6$
$P_2P_3P_4P_5$ $P_2P_3P_5P_6$ $\beta = (\frac{17}{56}, \frac{11}{56}, \frac{11}{56}, \frac{7}{56}, \frac{7}{56}, \frac{3}{56})$

$0.5c.14$ $P_1P_4P_5P_6$ $P_2P_3P_5P_6$
$P_2P_3P_4P_5$ $P_2P_4P_5P_6$ $\beta = (\frac{16}{58}, \frac{12}{58}, \frac{10}{58}, \frac{8}{58}, \frac{8}{58}, \frac{4}{58})$
$P_2P_3P_4P_6$

$0.5c.15$ $P_1P_4P_5P_6$ $P_2P_3P_5P_6$
$P_2P_3P_4P_5$ $P_2P_4P_5P_6$ $\beta = (\frac{15}{60}, \frac{11}{60}, \frac{11}{60}, \frac{9}{60}, \frac{9}{60}, \frac{5}{60})$
$P_2P_3P_4P_6$ $P_3P_4P_5P_6$

$0.6a.11$ $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_3P_4$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_3P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_2P_3P_4$ $P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$ $\beta = (\frac{16}{52}, \frac{12}{52}, \frac{12}{52}, \frac{8}{52}, \frac{4}{52}, 0)$
$P_1P_3P_4P_5$
$P_1P_3P_4P_6$
$P_1P_3P_5P_6$
$P_2P_3P_4P_5$
$P_2P_3P_4P_6$

$0.6a.12a$ $P_1P_4P_5P_6$ $\beta = (\frac{17}{54}, \frac{11}{54}, \frac{11}{54}, \frac{9}{54}, \frac{5}{54}, \frac{1}{54})$

0.6a.12b $P_2P_3P_5P_6$ $\beta = (\frac{15}{54}, \frac{13}{54}, \frac{13}{54}, \frac{7}{54}, \frac{5}{54}, \frac{1}{54})$

0.6a.13 $\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_5P_6 \end{array}$ $\beta = (\frac{16}{56}, \frac{12}{56}, \frac{12}{56}, \frac{8}{56}, \frac{6}{56}, \frac{2}{56})$

0.6a.14 $\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_5P_6 \\ P_2P_4P_5P_6 \end{array}$ $\beta = (\frac{15}{58}, \frac{13}{58}, \frac{11}{58}, \frac{9}{58}, \frac{7}{58}, \frac{3}{58})$

0.6a.15 $\begin{array}{ll} P_1P_4P_5P_6 & P_2P_4P_5P_6 \\ P_2P_3P_5P_6 & P_3P_4P_5P_6 \end{array}$ $\beta = (\frac{14}{60}, \frac{12}{60}, \frac{12}{60}, \frac{10}{60}, \frac{8}{60}, \frac{4}{60})$

0.6b.9 $\begin{array}{lll} P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_4P_5 & \\ & P_1P_3P_4P_6 & \\ & P_1P_3P_5P_6 & \end{array}$ $\beta = (\frac{20}{48}, \frac{10}{48}, \frac{8}{48}, \frac{4}{48}, \frac{4}{48}, \frac{2}{48})$

0.6b.10a $P_1P_4P_5P_6$ $\beta = (\frac{21}{50}, \frac{9}{50}, \frac{7}{50}, \frac{5}{50}, \frac{5}{50}, \frac{3}{50})$

0.6b.10b $P_2P_3P_4P_5$ $\beta = (\frac{19}{50}, \frac{11}{50}, \frac{9}{50}, \frac{5}{50}, \frac{5}{50}, \frac{1}{50})$

0.6b.11a $\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \end{array}$ $\beta = (\frac{20}{52}, \frac{10}{52}, \frac{8}{52}, \frac{6}{52}, \frac{6}{52}, \frac{2}{52})$

0.6b.11b $\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array}$ $\beta = (\frac{18}{52}, \frac{12}{52}, \frac{10}{52}, \frac{6}{52}, \frac{4}{52}, \frac{2}{52})$

0.6b.12a $\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array}$ $\beta = (\frac{19}{54}, \frac{11}{54}, \frac{9}{54}, \frac{7}{54}, \frac{5}{54}, \frac{3}{54})$

0.6b.12b $\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array}$ $\beta = (\frac{17}{54}, \frac{13}{54}, \frac{11}{54}, \frac{5}{54}, \frac{5}{54}, \frac{3}{54})$

0.6b.13 $\begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_6 \\ P_2P_3P_4P_5 & P_2P_3P_5P_6 \end{array}$ $\beta = (\frac{18}{56}, \frac{12}{56}, \frac{10}{56}, \frac{6}{56}, \frac{6}{56}, \frac{4}{56})$

0.6b.14 $\begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_6 \\ P_2P_4P_5P_6 & P_2P_3P_5P_6 \\ P_2P_3P_4P_5 & \end{array}$ $\beta = (\frac{17}{58}, \frac{13}{58}, \frac{9}{58}, \frac{7}{58}, \frac{7}{58}, \frac{5}{58})$

0.6b.15 $\begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_5 \\ P_2P_4P_5P_6 & P_2P_3P_4P_6 \\ P_3P_4P_5P_6 & P_2P_3P_5P_6 \end{array}$ $\beta = (\frac{16}{60}, \frac{12}{60}, \frac{10}{30}, \frac{8}{60}, \frac{8}{60}, \frac{6}{60})$

$0.6c.10$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{21}{50}, \frac{7}{50}, \frac{7}{50}, \frac{7}{50}, \frac{7}{50}, \frac{1}{50})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_4P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
		$P_1P_3P_4P_5$		
		$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		
		$P_1P_4P_5P_6$		

$0.6c.11$	$P_2P_3P_4P_5$	$\beta = (\frac{20}{52}, \frac{8}{52}, \frac{8}{52}, \frac{8}{52}, \frac{8}{52}, 0)$

$0.6c.12$	$P_2P_3P_4P_5$	$\beta = (\frac{19}{54}, \frac{9}{54}, \frac{9}{54}, \frac{9}{54}, \frac{7}{54}, \frac{1}{54})$
	$P_2P_3P_4P_6$	

$0.6c.13$	$P_2P_3P_4P_5$	$\beta = (\frac{18}{56}, \frac{10}{56}, \frac{10}{56}, \frac{8}{56}, \frac{8}{56}, \frac{2}{56})$
	$P_2P_3P_4P_6$	
	$P_2P_3P_5P_6$	

$0.6c.14$	$P_2P_3P_4P_5$	$P_2P_3P_4P_6$	$\beta = (\frac{17}{58}, \frac{11}{58}, \frac{9}{58}, \frac{9}{58}, \frac{9}{58}, \frac{3}{58})$
	$P_2P_3P_5P_6$	$P_2P_4P_5P_6$	

$0.6c.15$	$P_2P_3P_4P_5$	$P_2P_4P_5P_6$	$\beta = (\frac{16}{60}, \frac{10}{60}, \frac{10}{60}, \frac{10}{60}, \frac{10}{60}, \frac{4}{60})$
	$P_2P_3P_4P_6$	$P_3P_4P_5P_6$	
	$P_2P_3P_5P_6$		

$0.6d.10$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{15}{50}, \frac{15}{50}, \frac{9}{50}, \frac{9}{50}, \frac{1}{50}, \frac{1}{50})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_2P_3P_4$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
		$P_1P_3P_4P_5$		
		$P_1P_3P_4P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		

$0.6d.11$	$P_1P_3P_5P_6$	$\beta = (\frac{16}{52}, \frac{14}{52}, \frac{10}{52}, \frac{8}{52}, \frac{2}{52}, \frac{2}{52})$

$0.6d.12a$	$P_1P_3P_5P_6$	$\beta = (\frac{15}{54}, \frac{15}{54}, \frac{11}{54}, \frac{7}{54}, \frac{3}{54}, \frac{3}{54})$
	$P_2P_3P_5P_6$	

$0.6d.12b$	$P_1P_3P_5P_6$	$\beta = (\frac{17}{54}, \frac{13}{54}, \frac{9}{54}, \frac{9}{54}, \frac{3}{54}, \frac{3}{54})$
	$P_1P_4P_5P_6$	

$0.6d.13$	$P_1P_3P_5P_6$	$\beta = (\frac{16}{56}, \frac{14}{56}, \frac{10}{56}, \frac{8}{56}, \frac{4}{56}, \frac{4}{56})$
	$P_1P_4P_5P_6$	
	$P_2P_3P_5P_6$	

$0.6d.14$	$P_1P_3P_5P_6$	$P_2P_3P_5P_6$	$\beta = (\frac{15}{58}, \frac{15}{58}, \frac{9}{58}, \frac{9}{58}, \frac{5}{58}, \frac{5}{58})$
	$P_1P_4P_5P_6$	$P_2P_4P_5P_6$	

$0.6d.15$	$P_1P_3P_5P_6$	$P_2P_4P_5P_6$	$\beta = (\frac{14}{60}, \frac{14}{60}, \frac{10}{60}, \frac{10}{60}, \frac{6}{60}, \frac{6}{60})$
	$P_1P_4P_5P_6$	$P_3P_4P_5P_6$	
	$P_2P_3P_5P_6$		

$0.7a.9$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{21}{48}, \frac{9}{48}, \frac{9}{48}, \frac{3}{48}, \frac{3}{48}, \frac{3}{48})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		

$0.7a.10a$	$P_1P_4P_5P_6$	$\beta = (\frac{22}{50}, \frac{8}{50}, \frac{8}{50}, \frac{4}{50}, \frac{4}{50}, \frac{4}{50})$

$0.7a.10b$	$P_2P_3P_4P_5$	$\beta = (\frac{20}{50}, \frac{10}{50}, \frac{10}{50}, \frac{4}{50}, \frac{4}{50}, \frac{2}{50})$

$0.7a.11a$	$P_1P_4P_5P_6$	$\beta = (\frac{21}{52}, \frac{9}{52}, \frac{9}{52}, \frac{5}{52}, \frac{5}{52}, \frac{3}{52})$
	$P_2P_3P_4P_5$	

$0.7a.11b$	$P_2P_3P_4P_5$	$\beta = (\frac{19}{52}, \frac{11}{52}, \frac{11}{52}, \frac{5}{52}, \frac{3}{52}, \frac{3}{52})$
	$P_2P_3P_4P_6$	

$0.7a.12a$	$P_1P_4P_5P_6$	$\beta = (\frac{20}{54}, \frac{10}{54}, \frac{10}{54}, \frac{6}{54}, \frac{4}{54}, \frac{4}{54})$
	$P_2P_3P_4P_5$	
	$P_2P_3P_4P_6$	

$0.7a.12b$	$P_2P_3P_4P_5$	$\beta = (\frac{18}{54}, \frac{12}{54}, \frac{12}{54}, \frac{4}{54}, \frac{4}{54}, \frac{4}{54})$
	$P_2P_3P_4P_6$	
	$P_2P_3P_5P_6$	

$0.7a.13$	$P_1P_4P_5P_6$	$P_2P_3P_4P_5$	$\beta = (\frac{19}{56}, \frac{11}{56}, \frac{11}{56}, \frac{5}{56}, \frac{5}{56}, \frac{5}{56})$
	$P_2P_3P_5P_6$	$P_2P_3P_4P_6$	

$0.7a.14$	$P_1P_4P_5P_6$	$P_2P_3P_5P_6$	$\beta = (\frac{18}{58}, \frac{12}{58}, \frac{10}{58}, \frac{6}{58}, \frac{6}{58}, \frac{6}{58})$
	$P_2P_3P_4P_5$	$P_2P_4P_5P_6$	
	$P_2P_3P_4P_6$		

$0.7a.15$	$P_1P_4P_5P_6$	$P_2P_3P_5P_6$	$\beta = (\frac{17}{60}, \frac{11}{60}, \frac{11}{60}, \frac{7}{60}, \frac{7}{60}, \frac{7}{60})$
	$P_2P_3P_4P_5$	$P_2P_4P_5P_6$	
	$P_2P_3P_4P_6$	$P_3P_4P_5P_6$	

$0.7b.11$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{17}{52}, \frac{13}{52}, \frac{11}{52}, \frac{7}{52}, \frac{3}{52}, \frac{1}{52})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_2P_3P_4$	$P_1P_3P_4P_5$		
		$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		

$0.7b.12a$	$P_2P_3P_5P_6$	$\beta = (\frac{16}{54}, \frac{14}{54}, \frac{12}{54}, \frac{6}{54}, \frac{4}{54}, \frac{2}{54})$

$0.7b.12b$	$P_1P_4P_5P_6$	$\beta = (\frac{18}{54}, \frac{12}{54}, \frac{10}{54}, \frac{8}{54}, \frac{4}{54}, \frac{2}{54})$

$0.7b.13$	$P_1P_4P_5P_6$	$\beta = (\frac{17}{56}, \frac{13}{56}, \frac{11}{56}, \frac{7}{56}, \frac{5}{56}, \frac{3}{56})$
	$P_2P_3P_5P_6$	

0.7b.14 $P_1P_4P_5P_6$
$P_2P_3P_5P_6$
$P_2P_4P_5P_6$
$\beta = (\frac{16}{58}, \frac{14}{58}, \frac{10}{58}, \frac{8}{58}, \frac{6}{58}, \frac{4}{58})$

0.7b.15 $P_1P_4P_5P_6$ $P_2P_4P_5P_6$
$P_2P_3P_5P_6$ $P_3P_4P_5P_6$
$\beta = (\frac{15}{60}, \frac{13}{60}, \frac{11}{60}, \frac{9}{60}, \frac{7}{60}, \frac{5}{60})$

0.7c.12 $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_3P_4$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_3P_5$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_1P_4P_5$ $P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$
$P_2P_3P_4$ $P_1P_3P_4P_5$
$P_1P_3P_4P_6$
$P_1P_3P_5P_6$
$P_1P_4P_5P_6$
$P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$\beta = (\frac{18}{54}, \frac{10}{54}, \frac{10}{54}, \frac{10}{54}, \frac{6}{54}, 0)$

0.7c.13 $P_2P_3P_5P_6$
$\beta = (\frac{17}{56}, \frac{11}{56}, \frac{11}{56}, \frac{9}{56}, \frac{7}{56}, \frac{1}{56})$

0.7c.14 $P_2P_3P_5P_6$
$P_2P_4P_5P_6$
$\beta = (\frac{16}{58}, \frac{12}{58}, \frac{10}{58}, \frac{10}{58}, \frac{8}{58}, \frac{2}{58})$

0.7c.15 $P_2P_3P_5P_6$
$P_2P_4P_5P_6$
$P_3P_4P_5P_6$
$\beta = (\frac{15}{60}, \frac{11}{60}, \frac{11}{60}, \frac{11}{60}, \frac{9}{60}, \frac{3}{60})$

0.7d.10 $P_1P_2P_3$ $P_1P_2P_3P_4$ $P_1P_2P_3P_4P_5$
$P_1P_2P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_4P_6$
$P_1P_2P_5$ $P_1P_2P_3P_6$ $P_1P_2P_3P_5P_6$
$P_1P_2P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_5P_6$
$P_1P_3P_4$ $P_1P_2P_4P_6$ $P_1P_3P_4P_5P_6$
$P_1P_3P_5$ $P_1P_2P_5P_6$ $P_2P_3P_4P_5P_6$
$P_1P_4P_5$ $P_1P_3P_4P_5$
$P_1P_3P_4P_6$
$P_1P_3P_5P_6$
$P_1P_4P_5P_6$
$\beta = (\frac{22}{50}, \frac{8}{50}, \frac{6}{50}, \frac{6}{50}, \frac{6}{50}, \frac{2}{50})$

0.7d.11 $P_2P_3P_4P_5$
$\beta = (\frac{21}{52}, \frac{9}{52}, \frac{7}{52}, \frac{7}{52}, \frac{7}{52}, \frac{1}{52})$

0.7d.12 $P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$\beta = (\frac{20}{54}, \frac{10}{54}, \frac{8}{54}, \frac{8}{54}, \frac{6}{54}, \frac{2}{54})$

0.7d.13 $P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$P_2P_3P_5P_6$
$\beta = (\frac{19}{56}, \frac{11}{56}, \frac{9}{56}, \frac{7}{56}, \frac{7}{56}, \frac{3}{56})$

0.7d.14 $P_2P_3P_4P_5$ $P_2P_3P_5P_6$
$P_2P_3P_4P_6$ $P_2P_4P_5P_6$
$\beta = (\frac{18}{58}, \frac{12}{58}, \frac{8}{58}, \frac{8}{58}, \frac{8}{58}, \frac{4}{58})$

0.7d.15 $P_2P_3P_4P_5$ $P_2P_4P_5P_6$
$P_2P_3P_4P_6$ $P_3P_4P_5P_6$
$P_2P_3P_5P_6$
$\beta = (\frac{17}{60}, \frac{11}{60}, \frac{9}{60}, \frac{9}{60}, \frac{9}{60}, \frac{5}{60})$

$0.7e.12$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_2P_3P_4$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{14}{54}, \frac{14}{54}, \frac{14}{54}, \frac{6}{54}, \frac{6}{54}, 0)$
	$P_2P_3P_5$	$P_1P_3P_4P_5$		
		$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		
		$P_2P_3P_5P_6$		

$0.7e.13$	$P_1P_4P_5P_6$	$\beta = (\frac{15}{56}, \frac{13}{56}, \frac{13}{56}, \frac{7}{56}, \frac{7}{56}, \frac{1}{56})$
$0.7e.14$	$P_1P_4P_5P_6$	$\beta = (\frac{14}{58}, \frac{14}{58}, \frac{12}{58}, \frac{8}{58}, \frac{8}{58}, \frac{2}{58})$
	$P_2P_4P_5P_6$	
$0.7e.15$	$P_1P_4P_5P_6$	
	$P_2P_4P_5P_6$	$\beta = (\frac{13}{60}, \frac{13}{60}, \frac{13}{60}, \frac{9}{60}, \frac{9}{60}, \frac{3}{60})$
	$P_3P_4P_5P_6$	

$0.8a.12$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{15}{54}, \frac{15}{54}, \frac{13}{54}, \frac{5}{54}, \frac{5}{54}, \frac{1}{54})$
	$P_2P_3P_4$	$P_1P_3P_4P_5$		
	$P_2P_3P_5$	$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		
		$P_2P_3P_5P_6$		

$0.8a.13$	$P_1P_4P_5P_6$	$\beta = (\frac{16}{56}, \frac{14}{56}, \frac{12}{56}, \frac{6}{56}, \frac{6}{56}, \frac{2}{56})$
$0.8a.14$	$P_1P_4P_5P_6$	$\beta = (\frac{15}{58}, \frac{15}{58}, \frac{11}{58}, \frac{7}{58}, \frac{7}{58}, \frac{3}{58})$
	$P_2P_4P_5P_6$	
$0.8a.15$	$P_1P_4P_5P_6$	
	$P_2P_4P_5P_6$	$\beta = (\frac{14}{60}, \frac{14}{60}, \frac{12}{60}, \frac{8}{60}, \frac{8}{60}, \frac{4}{60})$
	$P_3P_4P_5P_6$	

$0.8b.11$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{18}{52}, \frac{12}{52}, \frac{12}{52}, \frac{6}{52}, \frac{2}{52}, \frac{2}{52})$
	$P_1P_3P_6$	$P_1P_3P_4P_5$		
	$P_2P_3P_4$	$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		

$0.8b.12a$	$P_1P_4P_5P_6$	$\beta = (\frac{19}{54}, \frac{11}{54}, \frac{11}{54}, \frac{7}{54}, \frac{3}{54}, \frac{3}{54})$
$0.8b.12b$	$P_2P_3P_5P_6$	$\beta = (\frac{17}{54}, \frac{13}{54}, \frac{13}{54}, \frac{5}{54}, \frac{3}{54}, \frac{3}{54})$

0.8b.13	$P_1P_4P_5P_6$, $P_2P_3P_5P_6$	$\beta = (\frac{18}{56}, \frac{12}{56}, \frac{12}{56}, \frac{6}{56}, \frac{4}{56}, \frac{4}{56})$
0.8b.14	$P_1P_4P_5P_6$, $P_2P_3P_5P_6$, $P_2P_4P_5P_6$	$\beta = (\frac{17}{58}, \frac{13}{58}, \frac{11}{58}, \frac{7}{58}, \frac{5}{58}, \frac{5}{58})$
0.8b.15	$P_1P_4P_5P_6$, $P_3P_4P_5P_6$, $P_2P_3P_5P_6$, $P_2P_4P_5P_6$	$\beta = (\frac{16}{60}, \frac{12}{60}, \frac{12}{60}, \frac{8}{60}, \frac{6}{60}, \frac{6}{60})$
0.8c.12	$P_1P_2P_3$, $P_1P_2P_4$, $P_1P_2P_5$, $P_1P_2P_6$, $P_1P_3P_4$, $P_1P_3P_5$, $P_1P_4P_5$, $P_2P_3P_4$, $P_1P_2P_3P_4$, $P_1P_2P_3P_5$, $P_1P_2P_3P_6$, $P_1P_2P_4P_5$, $P_1P_2P_4P_6$, $P_1P_2P_5P_6$, $P_1P_3P_4P_5$, $P_1P_3P_4P_6$, $P_1P_3P_5P_6$, $P_1P_4P_5P_6$, $P_2P_3P_4P_5$, $P_2P_3P_4P_6$, $P_1P_2P_3P_4P_5$, $P_1P_2P_3P_4P_6$, $P_1P_2P_3P_5P_6$, $P_1P_2P_4P_5P_6$, $P_1P_3P_4P_5P_6$, $P_2P_3P_4P_5P_6$	$\beta = (\frac{19}{54}, \frac{11}{54}, \frac{9}{54}, \frac{9}{54}, \frac{5}{54}, \frac{1}{54})$
0.8c.13	$P_2P_3P_5P_6$	$\beta = (\frac{18}{56}, \frac{12}{56}, \frac{10}{56}, \frac{8}{56}, \frac{6}{56}, \frac{2}{56})$
0.8c.14	$P_2P_3P_5P_6$, $P_2P_4P_5P_6$	$\beta = (\frac{17}{58}, \frac{13}{58}, \frac{9}{58}, \frac{9}{58}, \frac{7}{58}, \frac{3}{58})$
0.8c.15	$P_1P_4P_5P_6$, $P_2P_3P_5P_6$	$\beta = (\frac{16}{60}, \frac{12}{60}, \frac{10}{60}, \frac{10}{60}, \frac{8}{60}, \frac{4}{60})$
0.8d.10	$P_1P_2P_3$, $P_1P_2P_4$, $P_1P_2P_5$, $P_1P_2P_6$, $P_1P_3P_4$, $P_1P_3P_5$, $P_1P_3P_6$, $P_1P_4P_5$, $P_1P_2P_3P_4$, $P_1P_2P_3P_5$, $P_1P_2P_3P_6$, $P_1P_2P_4P_5$, $P_1P_2P_4P_6$, $P_1P_2P_5P_6$, $P_1P_3P_4P_5$, $P_1P_3P_4P_6$, $P_1P_3P_5P_6$, $P_1P_4P_5P_6$, $P_1P_2P_3P_4P_5$, $P_1P_2P_3P_4P_6$, $P_1P_2P_3P_5P_6$, $P_1P_2P_4P_5P_6$, $P_1P_3P_4P_5P_6$, $P_2P_3P_4P_5P_6$	$\beta = (\frac{23}{50}, \frac{7}{50}, \frac{7}{50}, \frac{5}{50}, \frac{5}{50}, \frac{3}{50})$
0.8d.11	$P_2P_3P_4P_5$	$\beta = (\frac{22}{52}, \frac{8}{52}, \frac{8}{52}, \frac{6}{52}, \frac{6}{52}, \frac{2}{52})$
0.8d.12	$P_2P_3P_4P_5$, $P_2P_3P_4P_6$	$\beta = (\frac{21}{54}, \frac{9}{54}, \frac{9}{54}, \frac{7}{54}, \frac{5}{54}, \frac{3}{54})$
0.8d.13	$P_2P_3P_4P_5$, $P_2P_3P_4P_6$, $P_2P_3P_5P_6$	$\beta = (\frac{20}{56}, \frac{10}{56}, \frac{10}{56}, \frac{6}{56}, \frac{6}{56}, \frac{4}{56})$
0.8d.14	$P_2P_3P_4P_5$, $P_2P_3P_4P_6$, $P_2P_3P_5P_6$, $P_2P_4P_5P_6$	$\beta = (\frac{19}{58}, \frac{11}{58}, \frac{9}{58}, \frac{7}{58}, \frac{7}{58}, \frac{5}{58})$
0.8d.15	$P_2P_3P_4P_5$, $P_2P_3P_4P_6$, $P_2P_3P_5P_6$, $P_2P_4P_5P_6$, $P_3P_4P_5P_6$	$\beta = (\frac{18}{60}, \frac{10}{60}, \frac{10}{60}, \frac{8}{60}, \frac{8}{60}, \frac{6}{60})$

$0.8e.13$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_4P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_2P_3P_4$	$P_1P_3P_4P_5$		$\beta = (\frac{16}{56}, \frac{12}{56}, \frac{12}{56}, \frac{8}{56}, \frac{8}{56}, 0)$
	$P_2P_3P_5$	$P_1P_3P_4P_6$		
		$P_1P_3P_5P_6$		
		$P_1P_4P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		
		$P_2P_3P_5P_6$		
$0.8e.14$	$P_2P_4P_5P_6$	$\beta = (\frac{15}{58}, \frac{13}{58}, \frac{11}{58}, \frac{9}{58}, \frac{9}{58}, \frac{1}{58})$		
$0.8e.15$	$P_2P_4P_5P_6$	$\beta = (\frac{14}{60}, \frac{12}{60}, \frac{12}{60}, \frac{10}{60}, \frac{10}{60}, \frac{2}{60})$		
	$P_3P_4P_5P_6$			
$0.9a.14$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_4P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_2P_3P_4$	$P_1P_3P_4P_5$		$\beta = (\frac{14}{58}, \frac{14}{58}, \frac{10}{58}, \frac{10}{58}, \frac{10}{58}, 0)$
	$P_2P_3P_5$	$P_1P_3P_4P_6$		
	$P_2P_4P_5$	$P_1P_3P_5P_6$		
		$P_1P_4P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		
		$P_2P_3P_5P_6$		
		$P_2P_4P_5P_6$		
$0.9a.15$	$P_3P_4P_5P_6$	$\beta = (\frac{13}{60}, \frac{13}{60}, \frac{11}{60}, \frac{11}{60}, \frac{11}{60}, \frac{1}{60})$		
$0.9b.12$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{16}{54}, \frac{14}{54}, \frac{14}{54}, \frac{4}{54}, \frac{4}{54}, \frac{2}{54})$
	$P_1P_3P_6$	$P_1P_3P_4P_5$		
	$P_2P_3P_4$	$P_1P_3P_4P_6$		
	$P_2P_3P_5$	$P_1P_3P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		
		$P_2P_3P_5P_6$		
$0.9b.13$	$P_1P_4P_5P_6$	$\beta = (\frac{17}{56}, \frac{13}{56}, \frac{13}{56}, \frac{5}{56}, \frac{5}{56}, \frac{3}{56})$		
$0.9b.14$	$P_1P_4P_5P_6$	$\beta = (\frac{16}{58}, \frac{14}{58}, \frac{12}{58}, \frac{6}{58}, \frac{6}{58}, \frac{4}{58})$		
	$P_2P_4P_5P_6$			
$0.9b.15$	$P_1P_4P_5P_6$	$\beta = (\frac{15}{60}, \frac{13}{60}, \frac{13}{60}, \frac{7}{60}, \frac{7}{60}, \frac{5}{60})$		
$P_2P_4P_5P_6$	$P_3P_4P_5P_6$			

$0.9c.12$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{20}{54}, \frac{10}{54}, \frac{10}{54}, \frac{8}{54}, \frac{4}{54}, \frac{2}{54})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_6$	$P_1P_3P_4P_5$		
	$P_1P_4P_5$	$P_1P_3P_4P_6$		
	$P_2P_3P_4$	$P_1P_3P_5P_6$		
		$P_1P_4P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		

$0.9c.13$	$P_2P_3P_5P_6$	$\beta = (\frac{19}{56}, \frac{11}{56}, \frac{11}{56}, \frac{7}{56}, \frac{5}{56}, \frac{3}{56})$

$0.9c.14$	$P_2P_3P_5P_6$	$\beta = (\frac{18}{58}, \frac{12}{58}, \frac{10}{58}, \frac{8}{58}, \frac{6}{58}, \frac{4}{58})$
	$P_2P_4P_5P_6$	

$0.9c.15$	$P_2P_3P_5P_6$	$\beta = (\frac{17}{60}, \frac{11}{60}, \frac{11}{60}, \frac{9}{60}, \frac{7}{60}, \frac{5}{60})$
	$P_2P_4P_5P_6$	
	$P_3P_4P_5P_6$	

$0.9d.13$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{17}{56}, \frac{13}{56}, \frac{11}{56}, \frac{7}{56}, \frac{7}{56}, \frac{1}{56})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_4P_5$	$P_1P_3P_4P_5$		
	$P_2P_3P_4$	$P_1P_3P_4P_6$		
	$P_2P_3P_5$	$P_1P_3P_5P_6$		
		$P_1P_4P_5P_6$		
		$P_2P_3P_4P_5$		
		$P_2P_3P_4P_6$		
		$P_2P_3P_5P_6$		

$0.9d.14$	$P_2P_4P_5P_6$	$\beta = (\frac{16}{58}, \frac{14}{58}, \frac{10}{58}, \frac{8}{58}, \frac{8}{58}, \frac{2}{58})$

$0.9d.15$	$P_2P_4P_5P_6$	$\beta = (\frac{15}{60}, \frac{13}{60}, \frac{11}{60}, \frac{9}{60}, \frac{9}{60}, \frac{3}{60})$
	$P_3P_4P_5P_6$	

$0.9e.10$	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	$\beta = (\frac{24}{50}, \frac{6}{50}, \frac{6}{50}, \frac{6}{50}, \frac{4}{50}, \frac{4}{50})$
	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
	$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
	$P_1P_3P_6$	$P_1P_3P_4P_5$		
	$P_1P_4P_5$	$P_1P_3P_4P_6$		
	$P_1P_4P_6$	$P_1P_3P_5P_6$		
		$P_1P_4P_5P_6$		

$0.9e.11$	$P_2P_3P_4P_5$	$\beta = (\frac{23}{52}, \frac{7}{52}, \frac{7}{52}, \frac{7}{52}, \frac{5}{52}, \frac{3}{52})$

$0.9e.12$	$P_2P_3P_4P_5$	$\beta = (\frac{22}{54}, \frac{8}{54}, \frac{8}{54}, \frac{8}{54}, \frac{4}{54}, \frac{4}{54})$
	$P_2P_3P_4P_6$	

0.9e.13
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{21}{56}, \tfrac{9}{56}, \tfrac{9}{56}, \tfrac{7}{56}, \tfrac{5}{56}, \tfrac{5}{56})$$

0.9e.14
$$\begin{array}{ll} P_2P_3P_4P_5 & P_2P_3P_5P_6 \\ P_2P_3P_4P_6 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{20}{58}, \tfrac{10}{58}, \tfrac{8}{58}, \tfrac{8}{58}, \tfrac{6}{58}, \tfrac{6}{58})$$

0.9e.15
$$\begin{array}{ll} P_2P_3P_4P_5 & P_2P_4P_5P_6 \\ P_2P_3P_4P_6 & P_3P_4P_5P_6 \\ P_2P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{19}{60}, \tfrac{9}{60}, \tfrac{9}{60}, \tfrac{9}{60}, \tfrac{7}{60}, \tfrac{7}{60})$$

0.10a.13
$$\begin{array}{lll} P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ P_1P_3P_6 & P_1P_3P_4P_5 & \\ P_1P_4P_5 & P_1P_3P_4P_6 & \\ P_2P_3P_4 & P_1P_3P_5P_6 & \\ P_2P_3P_5 & P_1P_4P_5P_6 & \\ & P_2P_3P_4P_5 & \\ & P_2P_3P_4P_6 & \\ & P_2P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{18}{56}, \tfrac{12}{56}, \tfrac{12}{56}, \tfrac{6}{56}, \tfrac{6}{56}, \tfrac{2}{56})$$

0.10a.14
$$P_2P_4P_5P_6 \qquad \beta = (\tfrac{17}{58}, \tfrac{13}{58}, \tfrac{11}{58}, \tfrac{7}{58}, \tfrac{7}{58}, \tfrac{3}{58})$$

0.10a.15
$$\begin{array}{l} P_2P_4P_5P_6 \\ P_3P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{60}, \tfrac{12}{60}, \tfrac{12}{60}, \tfrac{8}{60}, \tfrac{8}{60}, \tfrac{4}{60})$$

0.10b.14
$$\begin{array}{lll} P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ P_1P_4P_5 & P_2P_3P_5P_6 & \\ P_2P_3P_4 & P_2P_4P_5P_6 & \\ P_2P_3P_5 & P_1P_3P_4P_5 & \\ P_2P_4P_5 & P_1P_3P_4P_6 & \\ & P_1P_3P_5P_6 & \\ & P_1P_4P_5P_6 & \\ & P_2P_3P_4P_5 & \\ & P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{15}{58}, \tfrac{15}{58}, \tfrac{9}{58}, \tfrac{9}{58}, \tfrac{9}{58}, \tfrac{1}{58})$$

0.10b.15
$$P_3P_4P_5P_6 \qquad \beta = (\tfrac{14}{60}, \tfrac{14}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{2}{60})$$

0.10c.10
$$\begin{array}{lll} P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ P_1P_3P_6 & P_1P_3P_4P_5 & \\ P_1P_4P_5 & P_1P_3P_4P_6 & \\ P_1P_4P_6 & P_1P_3P_5P_6 & \\ P_1P_5P_6 & P_1P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{25}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{5}{50})$$

0.10c.11
$$P_2P_3P_4P_5 \quad \beta = (\tfrac{24}{52}, \tfrac{6}{52}, \tfrac{6}{52}, \tfrac{6}{52}, \tfrac{6}{52}, \tfrac{4}{52})$$

0.10c.12

$P_2P_3P_4P_5$
$P_2P_3P_4P_6$

$$\beta = (\tfrac{23}{54}, \tfrac{7}{54}, \tfrac{7}{54}, \tfrac{7}{54}, \tfrac{5}{54}, \tfrac{5}{54})$$

0.10c.13

$P_2P_3P_4P_5$
$P_2P_3P_4P_6$
$P_2P_3P_5P_6$

$$\beta = (\tfrac{22}{56}, \tfrac{8}{56}, \tfrac{8}{56}, \tfrac{6}{56}, \tfrac{6}{56}, \tfrac{6}{56})$$

0.10c.14

$P_2P_3P_4P_5$	$P_2P_3P_5P_6$
$P_2P_3P_4P_6$	$P_2P_4P_5P_6$

$$\beta = (\tfrac{21}{58}, \tfrac{9}{58}, \tfrac{7}{58}, \tfrac{7}{58}, \tfrac{7}{58}, \tfrac{7}{58})$$

0.10c.15

$P_2P_3P_4P_5$	$P_2P_4P_5P_6$
$P_2P_3P_4P_6$	$P_3P_4P_5P_6$
$P_2P_3P_5P_6$	

$$\beta = (\tfrac{20}{60}, \tfrac{8}{60}, \tfrac{8}{60}, \tfrac{8}{60}, \tfrac{8}{60}, \tfrac{8}{60})$$

0.10d.12

$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
$P_1P_3P_6$	$P_1P_3P_4P_5$	
$P_1P_4P_5$	$P_1P_3P_4P_6$	
$P_1P_4P_6$	$P_1P_3P_5P_6$	
$P_2P_3P_4$	$P_1P_4P_5P_6$	
	$P_2P_3P_4P_5$	
	$P_2P_3P_4P_6$	

$$\beta = (\tfrac{21}{54}, \tfrac{9}{54}, \tfrac{9}{54}, \tfrac{9}{54}, \tfrac{3}{54}, \tfrac{3}{54})$$

0.10d.13

$P_2P_3P_5P_6$

$$\beta = (\tfrac{20}{56}, \tfrac{10}{56}, \tfrac{10}{56}, \tfrac{8}{56}, \tfrac{4}{56}, \tfrac{4}{56})$$

0.10d.14

$P_2P_3P_5P_6$
$P_2P_4P_5P_6$

$$\beta = (\tfrac{19}{58}, \tfrac{11}{58}, \tfrac{9}{58}, \tfrac{9}{58}, \tfrac{5}{58}, \tfrac{5}{58})$$

0.10d.15

$P_2P_3P_5P_6$
$P_2P_4P_5P_6$
$P_3P_4P_5P_6$

$$\beta = (\tfrac{18}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{10}{60}, \tfrac{6}{60}, \tfrac{6}{60})$$

0.10e.15

$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
$P_1P_3P_4$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
$P_1P_3P_5$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
$P_1P_4P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
$P_2P_3P_4$	$P_1P_3P_4P_5$	
$P_2P_3P_5$	$P_1P_3P_4P_6$	
$P_2P_4P_5$	$P_1P_3P_5P_6$	
$P_3P_4P_5$	$P_1P_4P_5P_6$	
	$P_2P_3P_4P_5$	
	$P_2P_3P_4P_6$	
	$P_2P_3P_5P_6$	
	$P_2P_4P_5P_6$	
	$P_3P_4P_5P_6$	

$$\beta = (\tfrac{12}{60}, \tfrac{12}{60}, \tfrac{12}{60}, \tfrac{12}{60}, \tfrac{12}{60}, 0)$$

$$\begin{array}{llll} 0.10f.12 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & P_2P_3P_4 & P_1P_3P_4P_6 & \\ & P_2P_3P_5 & P_1P_3P_5P_6 & \\ & P_2P_3P_6 & P_2P_3P_4P_5 & \\ & & P_2P_3P_4P_6 & \\ & & P_2P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{15}{54}, \tfrac{15}{54}, \tfrac{15}{54}, \tfrac{3}{54}, \tfrac{3}{54}, \tfrac{3}{54})$$

$$\begin{array}{ll} 0.10f.13 & P_1P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{56}, \tfrac{14}{56}, \tfrac{14}{56}, \tfrac{4}{56}, \tfrac{4}{56}, \tfrac{4}{56})$$

$$\begin{array}{ll} 0.10f.14 & P_1P_4P_5P_6 \\ & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{15}{58}, \tfrac{15}{58}, \tfrac{13}{58}, \tfrac{5}{58}, \tfrac{5}{58}, \tfrac{5}{58})$$

$$\begin{array}{ll} 0.10f.15 & P_1P_4P_5P_6 \\ & P_2P_4P_5P_6 \\ & P_3P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{14}{60}, \tfrac{14}{60}, \tfrac{14}{60}, \tfrac{6}{60}, \tfrac{6}{60}, \tfrac{6}{60})$$

$$\begin{array}{lllll} 1.4.6 & P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & & & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & & & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{40}, \tfrac{16}{40}, \tfrac{2}{40}, \tfrac{2}{40}, \tfrac{2}{40}, \tfrac{2}{40})$$

$$\begin{array}{ll} 1.4.7 & P_1P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{17}{42}, \tfrac{15}{42}, \tfrac{3}{42}, \tfrac{3}{42}, \tfrac{3}{42}, \tfrac{1}{42})$$

$$\begin{array}{ll} 1.4.8a & P_1P_3P_4P_5 \\ & P_1P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{18}{44}, \tfrac{14}{44}, \tfrac{4}{44}, \tfrac{4}{44}, \tfrac{2}{44}, \tfrac{2}{44})$$

$$\begin{array}{ll} 1.4.8b & P_1P_3P_4P_5 \\ & P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{16}{44}, \tfrac{16}{44}, \tfrac{4}{44}, \tfrac{4}{44}, \tfrac{4}{44}, \tfrac{0}{44})$$

$$\begin{array}{ll} 1.4.9a & P_1P_3P_4P_5 \\ & P_1P_3P_4P_6 \\ & P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{17}{46}, \tfrac{15}{46}, \tfrac{5}{46}, \tfrac{5}{46}, \tfrac{3}{46}, \tfrac{1}{46})$$

$$\begin{array}{ll} 1.4.9b & P_1P_3P_4P_5 \\ & P_1P_3P_4P_6 \\ & P_1P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{19}{46}, \tfrac{13}{46}, \tfrac{5}{46}, \tfrac{3}{46}, \tfrac{3}{46}, \tfrac{3}{46})$$

$$\begin{array}{lll} 1.4.10a & P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ & P_1P_3P_4P_6 & P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{16}{48}, \tfrac{16}{48}, \tfrac{6}{48}, \tfrac{6}{48}, \tfrac{2}{48}, \tfrac{2}{48})$$

$$\begin{array}{lll} 1.4.10b & P_1P_3P_4P_5 & P_1P_3P_5P_6 \\ & P_1P_3P_4P_6 & P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{18}{48}, \tfrac{14}{48}, \tfrac{6}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{2}{48})$$

$$\begin{array}{lll} 1.4.10c & P_1P_3P_4P_5 & P_1P_3P_5P_6 \\ & P_1P_3P_4P_6 & P_1P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{20}{48}, \tfrac{12}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{4}{48})$$

$$\begin{array}{lll} 1.4.11a & P_1P_3P_4P_5 & P_2P_3P_4P_5 \\ & P_1P_3P_4P_6 & P_2P_3P_4P_6 \\ & P_1P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{17}{50}, \tfrac{15}{50}, \tfrac{7}{50}, \tfrac{5}{50}, \tfrac{3}{50}, \tfrac{3}{50})$$

1.4.11b	$P_1P_3P_4P_5$ $P_1P_3P_4P_6$ $P_1P_3P_5P_6$	$P_1P_4P_5P_6$ $P_2P_3P_4P_5$		$\beta = (\frac{19}{50}, \frac{15}{50}, \frac{5}{50}, \frac{5}{50}, \frac{3}{50}, \frac{3}{50})$
1.4.12a	$P_1P_3P_4P_5$ $P_1P_3P_4P_6$ $P_1P_3P_5P_6$	$P_1P_4P_5P_6$ $P_2P_3P_4P_5$ $P_2P_3P_4P_6$		$\beta = (\frac{18}{52}, \frac{14}{52}, \frac{6}{52}, \frac{6}{52}, \frac{4}{52}, \frac{4}{52})$
1.4.12b	$P_1P_3P_4P_5$ $P_1P_3P_4P_6$ $P_1P_3P_5P_6$	$P_2P_3P_4P_5$ $P_2P_3P_4P_6$ $P_2P_3P_5P_6$		$\beta = (\frac{16}{52}, \frac{16}{52}, \frac{8}{52}, \frac{4}{52}, \frac{4}{52}, \frac{4}{52})$
1.4.13	$P_1P_3P_4P_5$ $P_1P_3P_4P_6$ $P_1P_3P_5P_6$ $P_1P_4P_5P_6$	$P_2P_3P_4P_5$ $P_2P_3P_4P_6$ $P_2P_3P_5P_6$		$\beta = (\frac{17}{54}, \frac{15}{54}, \frac{7}{54}, \frac{5}{54}, \frac{5}{54}, \frac{5}{54})$
1.4.14	$P_1P_3P_4P_5$ $P_1P_3P_4P_6$ $P_1P_3P_5P_6$ $P_1P_4P_5P_6$	$P_2P_3P_4P_5$ $P_2P_3P_4P_6$ $P_2P_3P_5P_6$ $P_2P_4P_5P_6$		$\beta = (\frac{16}{56}, \frac{16}{56}, \frac{6}{56}, \frac{6}{56}, \frac{6}{56}, \frac{6}{56})$
1.5.7	P_1P_2	$P_1P_2P_3$ $P_1P_2P_4$ $P_1P_2P_5$ $P_1P_2P_6$ $P_1P_3P_4$	$P_1P_2P_3P_4$ $P_1P_2P_3P_5$ $P_1P_2P_3P_6$ $P_1P_2P_4P_5$ $P_1P_2P_4P_6$ $P_1P_2P_5P_6$ $P_1P_3P_4P_6$	$P_1P_2P_3P_4P_5$ $P_1P_2P_3P_4P_6$ $P_1P_2P_3P_5P_6$ $P_1P_2P_4P_5P_6$ $P_1P_3P_4P_5P_6$ $P_2P_3P_4P_5P_6$ $\beta = (\frac{19}{44}, \frac{13}{44}, \frac{5}{44}, \frac{5}{44}, \frac{1}{44}, \frac{1}{44})$
1.5.8a	$P_2P_3P_4P_5$			$\beta = (\frac{18}{46}, \frac{14}{46}, \frac{6}{46}, \frac{6}{46}, \frac{2}{46}, 0)$
1.5.8b	$P_1P_3P_5P_6$			$\beta = (\frac{20}{46}, \frac{12}{46}, \frac{6}{46}, \frac{4}{46}, \frac{2}{46}, \frac{2}{46})$
1.5.9a	$P_2P_3P_4P_5$ $P_2P_3P_4P_6$			$\beta = (\frac{17}{48}, \frac{15}{48}, \frac{7}{48}, \frac{7}{48}, \frac{1}{48}, \frac{1}{48})$
1.5.9b	$P_1P_3P_5P_6$ $P_2P_3P_4P_5$			$\beta = (\frac{19}{48}, \frac{13}{48}, \frac{7}{48}, \frac{5}{48}, \frac{3}{48}, \frac{1}{48})$
1.5.9c	$P_1P_3P_5P_6$ $P_1P_4P_5P_6$			$\beta = (\frac{21}{48}, \frac{11}{48}, \frac{5}{48}, \frac{5}{48}, \frac{3}{48}, \frac{3}{48})$
1.5.10a	$P_1P_3P_5P_6$ $P_2P_3P_4P_5$ $P_2P_3P_4P_6$			$\beta = (\frac{18}{50}, \frac{14}{50}, \frac{8}{50}, \frac{6}{50}, \frac{2}{50}, \frac{2}{50})$
1.5.10b	$P_1P_3P_5P_6$ $P_1P_4P_5P_6$ $P_2P_3P_4P_5$			$\beta = (\frac{20}{50}, \frac{12}{50}, \frac{6}{50}, \frac{6}{50}, \frac{4}{50}, \frac{2}{50})$
1.5.11a	$P_1P_3P_5P_6$ $P_1P_4P_5P_6$	$P_2P_3P_4P_5$ $P_2P_3P_4P_6$		$\beta = (\frac{19}{52}, \frac{13}{52}, \frac{7}{52}, \frac{7}{52}, \frac{3}{52}, \frac{3}{52})$
1.5.11b	$P_1P_3P_5P_6$ $P_2P_3P_4P_5$	$P_2P_3P_4P_6$ $P_2P_3P_5P_6$		$\beta = (\frac{17}{52}, \frac{15}{52}, \frac{9}{52}, \frac{5}{52}, \frac{3}{52}, \frac{3}{52})$
1.5.12	$P_1P_3P_5P_6$ $P_1P_4P_5P_6$ $P_2P_3P_4P_5$	$P_2P_3P_4P_6$ $P_2P_3P_5P_6$		$\beta = (\frac{18}{54}, \frac{14}{54}, \frac{8}{54}, \frac{6}{54}, \frac{4}{54}, \frac{4}{54})$

$$
\begin{array}{lll}
1.5.13 & \begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_4P_5 \\ P_1P_4P_5P_6 & P_2P_3P_4P_6 \\ P_2P_4P_5P_6 & P_2P_3P_5P_6 \end{array} & \beta = (\frac{17}{56}, \frac{15}{56}, \frac{7}{56}, \frac{7}{56}, \frac{5}{56}, \frac{5}{56})
\end{array}
$$

$$
\begin{array}{lllllll}
1.6a.9 & P_1P_2 & \begin{array}{c} P_1P_2P_3 \\ P_1P_2P_4 \\ P_1P_2P_5 \\ P_1P_2P_6 \\ P_1P_3P_4 \\ P_1P_3P_5 \\ P_1P_3P_4P_5 \\ P_1P_3P_4P_6 \\ P_1P_3P_5P_6 \end{array} & \begin{array}{c} P_1P_2P_3P_4 \\ P_1P_2P_3P_5 \\ P_1P_2P_3P_6 \\ P_1P_2P_4P_5 \\ P_1P_2P_4P_6 \\ P_1P_2P_5P_6 \end{array} & \begin{array}{c} P_1P_2P_3P_4P_5 \\ P_1P_2P_3P_4P_6 \\ P_1P_2P_3P_5P_6 \\ P_1P_2P_4P_5P_6 \\ P_1P_3P_4P_5P_6 \\ P_2P_3P_4P_5P_6 \end{array} & \beta = (\frac{21}{46}, \frac{11}{46}, \frac{7}{46}, \frac{3}{46}, \frac{3}{46}, \frac{1}{46})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.10a & P_2P_3P_4P_5 & \beta = (\frac{20}{48}, \frac{12}{48}, \frac{8}{48}, \frac{4}{48}, \frac{4}{48}, \frac{0}{48})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.10b & P_1P_4P_5P_6 & \beta = (\frac{22}{48}, \frac{10}{48}, \frac{6}{48}, \frac{4}{48}, \frac{4}{48}, \frac{2}{48})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.11a & \begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} & \beta = (\frac{19}{50}, \frac{13}{50}, \frac{9}{50}, \frac{5}{50}, \frac{3}{50}, \frac{1}{50})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.11b & \begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \end{array} & \beta = (\frac{21}{50}, \frac{11}{50}, \frac{7}{50}, \frac{5}{50}, \frac{5}{50}, \frac{1}{50})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.12a & \begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} & \beta = (\frac{20}{52}, \frac{12}{52}, \frac{8}{52}, \frac{6}{52}, \frac{4}{52}, \frac{2}{52})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.12b & \begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array} & \beta = (\frac{18}{52}, \frac{14}{52}, \frac{10}{52}, \frac{4}{52}, \frac{4}{52}, \frac{2}{52})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.13 & \begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_6 \\ P_2P_3P_4P_5 & P_2P_3P_5P_6 \end{array} & \beta = (\frac{19}{54}, \frac{13}{54}, \frac{9}{54}, \frac{5}{54}, \frac{5}{54}, \frac{3}{54})
\end{array}
$$

$$
\begin{array}{lll}
1.6a.14 & \begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_6 \\ P_2P_3P_4P_5 & P_2P_4P_5P_6 \\ P_2P_3P_5P_6 & \end{array} & \beta = (\frac{18}{56}, \frac{14}{56}, \frac{8}{56}, \frac{6}{56}, \frac{6}{56}, \frac{4}{56})
\end{array}
$$

$$
\begin{array}{llllll}
1.6b.10 & P_1P_2 & \begin{array}{c} P_1P_2P_3 \\ P_1P_2P_4 \\ P_1P_2P_5 \\ P_1P_2P_6 \\ P_1P_3P_4 \\ P_2P_3P_4 \end{array} & \begin{array}{c} P_1P_2P_3P_4 \\ P_1P_2P_3P_5 \\ P_1P_2P_3P_6 \\ P_1P_2P_4P_5 \\ P_1P_2P_4P_6 \\ P_1P_2P_5P_6 \\ P_1P_3P_4P_5 \\ P_1P_3P_4P_6 \\ P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} & \begin{array}{c} P_1P_2P_3P_4P_5 \\ P_1P_2P_3P_4P_6 \\ P_1P_2P_3P_5P_6 \\ P_1P_2P_4P_5P_6 \\ P_1P_3P_4P_5P_6 \\ P_2P_3P_4P_5P_6 \end{array} & \beta = (\frac{16}{48}, \frac{16}{48}, \frac{8}{48}, \frac{8}{48}, \frac{0}{48}, \frac{0}{48})
\end{array}
$$

$$
\begin{array}{lll}
1.6b.11 & P_1P_3P_5P_6 & \beta = (\frac{17}{50}, \frac{15}{50}, \frac{9}{50}, \frac{7}{50}, \frac{1}{50}, \frac{1}{50})
\end{array}
$$

$$
\begin{array}{lll}
1.6b.12a & \begin{array}{l} P_1P_3P_5P_6 \\ P_1P_4P_5P_6 \end{array} & \beta = (\frac{18}{52}, \frac{14}{52}, \frac{8}{52}, \frac{8}{52}, \frac{2}{52}, \frac{2}{52})
\end{array}
$$

$$
\begin{array}{lll}
1.6b.12b & \begin{array}{l} P_1P_3P_5P_6 \\ P_2P_3P_5P_6 \end{array} & \beta = (\frac{16}{52}, \frac{16}{52}, \frac{10}{52}, \frac{6}{52}, \frac{2}{52}, \frac{2}{52})
\end{array}
$$

1.6b.13
$$\begin{array}{l} P_1P_3P_5P_6 \\ P_1P_4P_5P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{17}{54}, \tfrac{15}{54}, \tfrac{9}{54}, \tfrac{7}{54}, \tfrac{3}{54}, \tfrac{3}{54})$$

1.6b.14
$$\begin{array}{ll} P_1P_3P_5P_6 & P_2P_3P_5P_6 \\ P_1P_4P_5P_6 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{16}{56}, \tfrac{16}{56}, \tfrac{8}{56}, \tfrac{8}{56}, \tfrac{4}{56}, \tfrac{4}{56})$$

1.7a.11
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_2P_3P_4 & P_1P_3P_4P_5 & \\ & & P_1P_3P_4P_6 & \\ & & P_1P_3P_5P_6 & \\ & & P_2P_3P_4P_5 & \\ & & P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{18}{50}, \tfrac{14}{50}, \tfrac{10}{50}, \tfrac{6}{50}, \tfrac{2}{50}, \tfrac{0}{50})$$

1.7a.12a $\quad P_1P_4P_5P_6 \qquad \beta = (\tfrac{19}{52}, \tfrac{13}{52}, \tfrac{9}{52}, \tfrac{7}{52}, \tfrac{3}{52}, \tfrac{1}{52})$

1.7a.12b $\quad P_2P_3P_5P_6 \qquad \beta = (\tfrac{17}{52}, \tfrac{15}{52}, \tfrac{11}{52}, \tfrac{5}{52}, \tfrac{3}{52}, \tfrac{1}{52})$

1.7a.13
$$\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{18}{54}, \tfrac{14}{54}, \tfrac{10}{54}, \tfrac{6}{54}, \tfrac{4}{54}, \tfrac{2}{54})$$

1.7a.14
$$\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_5P_6 \\ P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{17}{56}, \tfrac{15}{56}, \tfrac{9}{56}, \tfrac{7}{56}, \tfrac{5}{56}, \tfrac{3}{56})$$

1.7b.9
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & & P_1P_3P_4P_6 & \\ & & P_1P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{22}{46}, \tfrac{10}{46}, \tfrac{8}{46}, \tfrac{2}{46}, \tfrac{2}{46}, \tfrac{2}{46})$$

1.7b.10 $\quad P_2P_3P_4P_5 \qquad \beta = (\tfrac{21}{48}, \tfrac{11}{48}, \tfrac{9}{48}, \tfrac{3}{48}, \tfrac{3}{48}, \tfrac{1}{48})$

1.7b.11a
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{20}{50}, \tfrac{12}{50}, \tfrac{10}{50}, \tfrac{4}{50}, \tfrac{2}{50}, \tfrac{2}{50},)$$

1.7b.11b
$$\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{22}{50}, \tfrac{10}{50}, \tfrac{8}{50}, \tfrac{4}{50}, \tfrac{4}{50}, \tfrac{2}{50})$$

1.7b.12
$$\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{21}{52}, \tfrac{11}{52}, \tfrac{9}{52}, \tfrac{5}{52}, \tfrac{3}{52}, \tfrac{3}{52})$$

1.7b.13
$$\begin{array}{ll} P_2P_3P_4P_5 & P_2P_3P_5P_6 \\ P_2P_3P_4P_6 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{19}{52}, \tfrac{13}{52}, \tfrac{11}{52}, \tfrac{3}{52}, \tfrac{3}{52}, \tfrac{3}{52})$$

1.7b.13
$$\begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_5 \\ P_2P_3P_5P_6 & P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{20}{54}, \tfrac{12}{54}, \tfrac{10}{54}, \tfrac{4}{54}, \tfrac{4}{54}, \tfrac{4}{54})$$

1.7b.12
$$\begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_5P_6 \\ P_2P_3P_4P_5 & P_2P_4P_5P_6 \\ P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{19}{56}, \tfrac{13}{56}, \tfrac{9}{56}, \tfrac{5}{56}, \tfrac{5}{56}, \tfrac{5}{56})$$

1.7c.10
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_4P_5 & P_1P_3P_4P_5 & \\ & & P_1P_3P_4P_6 & \\ & & P_1P_3P_5P_6 & \\ & & P_1P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{23}{48}, \tfrac{9}{48}, \tfrac{5}{48}, \tfrac{5}{48}, \tfrac{5}{48}, \tfrac{1}{48})$$

1.7c.11
$$P_2P_3P_4P_5 \qquad \beta = (\tfrac{22}{50}, \tfrac{10}{50}, \tfrac{6}{50}, \tfrac{6}{50}, \tfrac{6}{50}, 0)$$

1.7c.12
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{21}{52}, \tfrac{11}{52}, \tfrac{7}{52}, \tfrac{7}{52}, \tfrac{5}{52}, \tfrac{1}{52})$$

1.7c.13
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{20}{54}, \tfrac{12}{54}, \tfrac{8}{54}, \tfrac{6}{54}, \tfrac{6}{54}, \tfrac{2}{54})$$

1.7c.14
$$\begin{array}{ll} P_2P_3P_4P_5 & P_2P_3P_5P_6 \\ P_2P_3P_4P_6 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{19}{56}, \tfrac{13}{56}, \tfrac{7}{56}, \tfrac{7}{56}, \tfrac{7}{56}, \tfrac{3}{56})$$

1.8a.10
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & P_1P_4P_5 & P_1P_3P_4P_6 & \\ & & P_1P_3P_5P_6 & \\ & & P_1P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{24}{48}, \tfrac{8}{48}, \tfrac{6}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{2}{48})$$

1.8a.11
$$P_2P_3P_4P_5 \qquad \beta = (\tfrac{23}{50}, \tfrac{9}{50}, \tfrac{7}{50}, \tfrac{5}{50}, \tfrac{5}{50}, \tfrac{1}{50})$$

1.8a.12
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{22}{52}, \tfrac{10}{52}, \tfrac{8}{52}, \tfrac{6}{52}, \tfrac{4}{52}, \tfrac{2}{52})$$

1.8a.13
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{21}{54}, \tfrac{11}{54}, \tfrac{9}{54}, \tfrac{5}{54}, \tfrac{5}{54}, \tfrac{3}{54})$$

1.8a.14
$$\begin{array}{ll} P_2P_3P_4P_5 & P_2P_3P_5P_6 \\ P_2P_3P_4P_6 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{20}{56}, \tfrac{12}{56}, \tfrac{8}{56}, \tfrac{6}{56}, \tfrac{6}{56}, \tfrac{4}{56})$$

1.8b.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{20}{52}, \frac{12}{52}, \frac{8}{52}, \frac{8}{52}, \frac{4}{52}, 0)$
		$P_1P_4P_5$	$P_1P_3P_4P_5$		
		$P_2P_3P_4$	$P_1P_3P_4P_6$		
			$P_1P_3P_5P_6$		
			$P_1P_4P_5P_6$		
			$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		

1.8b.13 $P_2P_3P_5P_6$ $\beta = (\frac{19}{54}, \frac{13}{54}, \frac{9}{54}, \frac{7}{54}, \frac{5}{54}, \frac{1}{54})$

1.8b.14 $P_2P_3P_5P_6$, $P_2P_4P_5P_6$ $\beta = (\frac{18}{56}, \frac{14}{56}, \frac{8}{56}, \frac{8}{56}, \frac{6}{56}, \frac{2}{56})$

1.8c.11	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{19}{50}, \frac{13}{50}, \frac{11}{50}, \frac{5}{50}, \frac{1}{50}, \frac{1}{50})$
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_2P_3P_4$	$P_1P_3P_4P_6$		
			$P_1P_3P_5P_6$		
			$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		

1.8c.12a $P_2P_3P_5P_6$ $\beta = (\frac{20}{52}, \frac{12}{52}, \frac{10}{52}, \frac{6}{52}, \frac{2}{52}, \frac{2}{52})$

1.8c.12b $P_1P_4P_5P_6$ $\beta = (\frac{18}{52}, \frac{14}{52}, \frac{12}{52}, \frac{4}{52}, \frac{2}{52}, \frac{2}{52})$

1.8c.13 $P_1P_4P_5P_6$, $P_2P_3P_5P_6$ $\beta = (\frac{19}{54}, \frac{13}{54}, \frac{11}{54}, \frac{5}{54}, \frac{3}{54}, \frac{3}{54})$

1.8c.14 $P_1P_4P_5P_6$, $P_2P_3P_5P_6$, $P_2P_4P_5P_6$ $\beta = (\frac{18}{56}, \frac{14}{56}, \frac{10}{56}, \frac{6}{56}, \frac{4}{56}, \frac{4}{56})$

1.8d.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{16}{52}, \frac{16}{52}, \frac{12}{52}, \frac{4}{52}, \frac{4}{52}, 0)$
		$P_2P_3P_4$	$P_1P_3P_4P_5$		
		$P_2P_3P_5$	$P_1P_3P_4P_6$		
			$P_1P_3P_5P_6$		
			$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		
			$P_2P_3P_5P_6$		

1.8d.13 $P_1P_4P_5P_6$ $\beta = (\frac{17}{54}, \frac{15}{54}, \frac{11}{54}, \frac{5}{54}, \frac{5}{54}, \frac{1}{54})$

1.8d.14 $P_1P_4P_5P_6$, $P_2P_4P_5P_6$ $\beta = (\frac{16}{56}, \frac{16}{56}, \frac{10}{56}, \frac{6}{56}, \frac{6}{56}, \frac{2}{56})$

1.9a.12
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & P_1P_4P_5 & P_1P_3P_4P_6 & \\ & P_2P_3P_4 & P_1P_3P_5P_6 & \\ & & P_1P_4P_5P_6 & \\ & & P_2P_3P_4P_5 & \\ & & P_2P_3P_4P_6 & \end{array} \qquad \beta = (\tfrac{21}{52}, \tfrac{11}{52}, \tfrac{9}{52}, \tfrac{7}{52}, \tfrac{3}{52}, \tfrac{1}{52})$$

1.9a.13 $P_2P_3P_5P_6$ $\quad \beta = (\tfrac{20}{54}, \tfrac{12}{54}, \tfrac{10}{54}, \tfrac{6}{54}, \tfrac{4}{54}, \tfrac{2}{54})$

1.9a.14
$$\begin{array}{l} P_2P_3P_5P_6 \\ P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{19}{56}, \tfrac{13}{56}, \tfrac{9}{56}, \tfrac{7}{56}, \tfrac{5}{56}, \tfrac{3}{56})$$

1.9b.10
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & P_1P_4P_5 & P_1P_3P_4P_6 & \\ & P_1P_4P_6 & P_1P_3P_5P_6 & \\ & & P_1P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{25}{48}, \tfrac{7}{48}, \tfrac{5}{48}, \tfrac{5}{48}, \tfrac{3}{48}, \tfrac{3}{48})$$

1.9b.11 $P_2P_3P_4P_5$ $\quad \beta = (\tfrac{24}{50}, \tfrac{8}{50}, \tfrac{6}{50}, \tfrac{6}{50}, \tfrac{4}{50}, \tfrac{2}{50})$

1.9b.12
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{23}{52}, \tfrac{9}{52}, \tfrac{7}{52}, \tfrac{7}{52}, \tfrac{3}{52}, \tfrac{3}{52})$$

1.9b.13
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{22}{54}, \tfrac{10}{54}, \tfrac{8}{54}, \tfrac{6}{54}, \tfrac{4}{54}, \tfrac{4}{54})$$

1.9b.14
$$\begin{array}{ll} P_2P_3P_4P_5 & P_2P_3P_5P_6 \\ P_2P_3P_4P_6 & P_2P_4P_5P_6 \end{array} \qquad \beta = (\tfrac{21}{56}, \tfrac{11}{56}, \tfrac{7}{56}, \tfrac{7}{56}, \tfrac{5}{56}, \tfrac{5}{56})$$

1.9c.13
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_3P_4P_5 & P_2P_3P_4P_5P_6 \\ & P_1P_4P_5 & P_1P_3P_4P_6 & \\ & P_2P_3P_4 & P_1P_3P_5P_6 & \\ & P_2P_3P_5 & P_1P_4P_5P_6 & \\ & & P_2P_3P_4P_5 & \\ & & P_1P_2P_5P_6 & \\ & & P_2P_3P_4P_6 & \\ & & P_2P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{18}{54}, \tfrac{14}{54}, \tfrac{10}{54}, \tfrac{6}{54}, \tfrac{6}{54}, 0)$$

1.9c.14 $P_2P_4P_5P_6$ $\quad \beta = (\tfrac{17}{56}, \tfrac{15}{56}, \tfrac{9}{56}, \tfrac{7}{56}, \tfrac{7}{56}, \tfrac{1}{56})$

1.9d.12

1.9d.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_2P_3P_4$	$P_1P_3P_4P_6$	
		$P_2P_3P_5$	$P_1P_3P_5P_6$	
			$P_2P_3P_4P_5$	
			$P_2P_3P_4P_6$	
			$P_2P_3P_5P_6$	

$$\beta = (\tfrac{17}{52}, \tfrac{15}{52}, \tfrac{13}{52}, \tfrac{3}{52}, \tfrac{3}{52}, \tfrac{1}{52})$$

1.9d.13 $P_1P_4P_5P_6$ $\quad \beta = (\frac{18}{54}, \frac{14}{54}, \frac{12}{54}, \frac{4}{54}, \frac{4}{54}, \frac{2}{54})$

1.9d.14 $P_1P_4P_5P_6$, $P_2P_4P_5P_6$ $\quad \beta = (\frac{17}{56}, \frac{15}{56}, \frac{11}{56}, \frac{5}{56}, \frac{5}{56}, \frac{3}{56})$

1.10a.14	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_4P_5$	$P_1P_3P_4P_5$	
		$P_2P_3P_4$	$P_1P_3P_4P_6$	
		$P_2P_3P_5$	$P_1P_3P_5P_6$	
		$P_2P_4P_5$	$P_1P_4P_5P_6$	
			$P_2P_3P_4P_5$	
			$P_2P_3P_4P_6$	
			$P_2P_3P_5P_6$	
			$P_2P_4P_5P_6$	

$$\beta = (\tfrac{16}{56}, \tfrac{16}{56}, \tfrac{8}{56}, \tfrac{8}{56}, \tfrac{8}{56}, 0)$$

1.10b.10	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_1P_4P_6$	$P_1P_3P_5P_6$	
		$P_1P_5P_6$	$P_1P_4P_5P_6$	

$$\beta = (\tfrac{26}{48}, \tfrac{6}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{4}{48}, \tfrac{4}{48})$$

1.10b.11 $P_2P_3P_4P_5$ $\quad \beta = (\frac{25}{50}, \frac{7}{50}, \frac{5}{50}, \frac{5}{50}, \frac{5}{50}, \frac{3}{50})$

1.10b.12 $P_2P_3P_4P_5$, $P_2P_3P_4P_6$ $\quad \beta = (\frac{24}{52}, \frac{8}{52}, \frac{6}{52}, \frac{6}{52}, \frac{4}{52}, \frac{4}{52})$

1.10b.13 $P_2P_3P_4P_5$, $P_2P_3P_4P_6$, $P_2P_3P_5P_6$ $\quad \beta = (\frac{23}{54}, \frac{9}{54}, \frac{7}{54}, \frac{5}{54}, \frac{5}{54}, \frac{5}{54})$

1.10b.14 $P_2P_3P_4P_5$, $P_2P_3P_4P_6$, $P_2P_3P_5P_6$, $P_2P_4P_5P_6$ $\quad \beta = (\frac{22}{56}, \frac{10}{56}, \frac{6}{56}, \frac{6}{56}, \frac{6}{56}, \frac{6}{56})$

1.10c.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{22}{52}, \frac{10}{52}, \frac{8}{52}, \frac{8}{52}, \frac{2}{52}, \frac{2}{52})$
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_1P_4P_5$	$P_1P_3P_4P_6$		
		$P_1P_4P_6$	$P_1P_3P_5P_6$		
		$P_2P_3P_4$	$P_1P_4P_5P_6$		
			$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		

1.10c.13	$P_2P_3P_5P_6$	$\beta = (\frac{21}{54}, \frac{11}{54}, \frac{9}{54}, \frac{7}{54}, \frac{3}{54}, \frac{3}{54})$

1.10c.14	$P_2P_3P_5P_6$	$\beta = (\frac{20}{56}, \frac{12}{56}, \frac{8}{56}, \frac{8}{56}, \frac{4}{56}, \frac{4}{56})$
	$P_2P_4P_5P_6$	

1.10d.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{16}{52}, \frac{16}{52}, \frac{14}{52}, \frac{2}{52}, \frac{2}{52}, \frac{2}{52})$
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_2P_3P_4$	$P_1P_3P_4P_6$		
		$P_2P_3P_5$	$P_1P_3P_5P_6$		
		$P_2P_3P_6$	$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		
			$P_2P_3P_5P_6$		

1.10d.13	$P_1P_4P_5P_6$	$\beta = (\frac{17}{54}, \frac{15}{54}, \frac{13}{54}, \frac{3}{54}, \frac{3}{54}, \frac{3}{54})$

1.10d.14	$P_1P_4P_5P_6$	$\beta = (\frac{16}{56}, \frac{16}{56}, \frac{12}{56}, \frac{4}{56}, \frac{4}{56}, \frac{4}{56})$
	$P_2P_4P_5P_6$	

1.10e.13	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
		$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
		$P_1P_3P_6$	$P_1P_3P_4P_5$		$\beta = (\frac{19}{54}, \frac{13}{54}, \frac{11}{54}, \frac{5}{54}, \frac{5}{54}, \frac{1}{54})$
		$P_1P_4P_5$	$P_1P_3P_4P_6$		
		$P_2P_3P_4$	$P_1P_3P_5P_6$		
		$P_2P_3P_5$	$P_1P_4P_5P_6$		
			$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		
			$P_2P_3P_5P_6$		

1.10e.14	$P_2P_4P_5P_6$	$\beta = (\frac{18}{56}, \frac{14}{56}, \frac{10}{56}, \frac{6}{56}, \frac{6}{56}, \frac{2}{56})$

2.7.9
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_3 & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & & P_1P_3P_4P_6 & \\ & & P_1P_3P_5P_6 & \end{array} \qquad \beta = (\tfrac{23}{44}, \tfrac{9}{44}, \tfrac{9}{44}, \tfrac{1}{44}, \tfrac{1}{44}, \tfrac{1}{44})$$

2.7.10a $P_1P_4P_5P_6$ $\quad \beta = (\frac{24}{46}, \frac{8}{46}, \frac{8}{46}, \frac{2}{46}, \frac{2}{46}, \frac{2}{46})$

2.7.10b $P_2P_3P_4P_5$ $\quad \beta = (\frac{22}{46}, \frac{10}{46}, \frac{10}{46}, \frac{2}{46}, \frac{2}{46}, 0)$

2.7.11a
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{21}{48}, \tfrac{11}{48}, \tfrac{11}{48}, \tfrac{3}{48}, \tfrac{1}{48}, \tfrac{1}{48})$$

2.7.11b
$$\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \end{array} \qquad \beta = (\tfrac{23}{48}, \tfrac{9}{48}, \tfrac{9}{48}, \tfrac{3}{48}, \tfrac{3}{48}, \tfrac{1}{48})$$

2.7.12a
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \\ P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{20}{50}, \tfrac{12}{50}, \tfrac{12}{50}, \tfrac{2}{50}, \tfrac{2}{50}, \tfrac{2}{50})$$

2.7.12b
$$\begin{array}{l} P_1P_4P_5P_6 \\ P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{22}{50}, \tfrac{10}{50}, \tfrac{10}{50}, \tfrac{4}{50}, \tfrac{2}{50}, \tfrac{2}{50})$$

2.7.13
$$\begin{array}{ll} P_1P_4P_5P_6 & P_2P_3P_4P_6 \\ P_2P_3P_4P_5 & P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{21}{52}, \tfrac{11}{52}, \tfrac{11}{52}, \tfrac{3}{52}, \tfrac{3}{52}, \tfrac{3}{52})$$

2.8a.10
$$\begin{array}{llll} P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\ P_1P_3 & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\ & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\ & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\ & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\ & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\ & P_1P_3P_6 & P_1P_3P_4P_5 & \\ & P_1P_4P_5 & P_1P_3P_4P_6 & \\ & & P_1P_3P_5P_6 & \\ & & P_1P_4P_5P_6 & \end{array} \qquad \beta = (\tfrac{25}{46}, \tfrac{7}{46}, \tfrac{7}{46}, \tfrac{3}{46}, \tfrac{3}{46}, \tfrac{1}{46})$$

2.8a.11 $P_2P_3P_4P_5$ $\quad \beta = (\frac{24}{48}, \frac{8}{48}, \frac{8}{48}, \frac{4}{48}, \frac{4}{48}, 0)$

2.8a.12
$$\begin{array}{l} P_2P_3P_4P_5 \\ P_2P_3P_4P_6 \end{array} \qquad \beta = (\tfrac{23}{50}, \tfrac{9}{50}, \tfrac{9}{50}, \tfrac{5}{50}, \tfrac{3}{50}, \tfrac{1}{50})$$

2.8a.13
$$\begin{array}{ll} & P_2P_3P_4P_5 \\ P_2P_3P_4P_6 & P_2P_3P_5P_6 \end{array} \qquad \beta = (\tfrac{22}{52}, \tfrac{10}{52}, \tfrac{10}{52}, \tfrac{4}{52}, \tfrac{4}{52}, \tfrac{2}{52})$$

$$\begin{array}{lllll}
2.8b.11 & P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\
 & P_1P_3 & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\
 & & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\
 & & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\
 & & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\
 & & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\
 & & P_1P_3P_6 & P_1P_3P_4P_5 & \\
 & & P_2P_3P_4 & P_1P_3P_4P_6 & \\
 & & & P_1P_3P_5P_6 & \\
 & & & P_2P_3P_4P_5 & \\
 & & & P_2P_3P_4P_6 &
\end{array}
\qquad \beta = (\tfrac{20}{48}, \tfrac{12}{48}, \tfrac{12}{48}, \tfrac{4}{48}, 0, 0)$$

$$2.8b.12a \quad P_1P_4P_5P_6 \qquad \beta = (\tfrac{21}{50}, \tfrac{11}{50}, \tfrac{11}{50}, \tfrac{5}{50}, \tfrac{1}{50}, \tfrac{1}{50})$$

$$2.8b.12b \quad P_2P_3P_5P_6 \qquad \beta = (\tfrac{19}{50}, \tfrac{13}{50}, \tfrac{13}{50}, \tfrac{3}{50}, \tfrac{1}{50}, \tfrac{1}{50})$$

$$\begin{array}{ll}
2.8b.13 & P_1P_4P_5P_6 \\
 & P_2P_3P_5P_6
\end{array}
\qquad \beta = (\tfrac{20}{52}, \tfrac{12}{52}, \tfrac{12}{52}, \tfrac{4}{52}, \tfrac{2}{52}, \tfrac{2}{52})$$

$$\begin{array}{lllll}
2.9a.10 & P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\
 & P_1P_3 & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\
 & & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\
 & & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\
 & & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\
 & & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\
 & & P_1P_3P_6 & P_1P_3P_4P_5 & \\
 & & P_1P_4P_5 & P_1P_3P_4P_6 & \\
 & & P_1P_4P_6 & P_1P_3P_5P_6 & \\
 & & & P_1P_4P_5P_6 &
\end{array}
\qquad \beta = (\tfrac{26}{46}, \tfrac{6}{46}, \tfrac{6}{46}, \tfrac{4}{46}, \tfrac{2}{46}, \tfrac{2}{46})$$

$$2.9a.11 \quad P_2P_3P_4P_5 \qquad \beta = (\tfrac{25}{48}, \tfrac{7}{48}, \tfrac{7}{48}, \tfrac{5}{48}, \tfrac{3}{48}, \tfrac{1}{48})$$

$$\begin{array}{ll}
2.9a.12 & P_2P_3P_4P_5 \\
 & P_2P_3P_4P_6
\end{array}
\qquad \beta = (\tfrac{24}{50}, \tfrac{8}{50}, \tfrac{8}{50}, \tfrac{6}{50}, \tfrac{2}{50}, \tfrac{2}{50})$$

$$\begin{array}{ll}
2.9a.13 & P_2P_3P_4P_5 \\
P_2P_3P_4P_6 & P_2P_3P_5P_6
\end{array}
\qquad \beta = (\tfrac{23}{52}, \tfrac{9}{52}, \tfrac{9}{52}, \tfrac{5}{52}, \tfrac{3}{52}, \tfrac{3}{52})$$

$$\begin{array}{lllll}
2.9b.12 & P_1P_2 & P_1P_2P_3 & P_1P_2P_3P_4 & P_1P_2P_3P_4P_5 \\
 & P_1P_3 & P_1P_2P_4 & P_1P_2P_3P_5 & P_1P_2P_3P_4P_6 \\
 & & P_1P_2P_5 & P_1P_2P_3P_6 & P_1P_2P_3P_5P_6 \\
 & & P_1P_2P_6 & P_1P_2P_4P_5 & P_1P_2P_4P_5P_6 \\
 & & P_1P_3P_4 & P_1P_2P_4P_6 & P_1P_3P_4P_5P_6 \\
 & & P_1P_3P_5 & P_1P_2P_5P_6 & P_2P_3P_4P_5P_6 \\
 & & P_1P_3P_6 & P_1P_3P_4P_5 & \\
 & & P_2P_3P_4 & P_1P_3P_4P_6 & \\
 & & P_2P_3P_5 & P_1P_3P_5P_6 & \\
 & & & P_2P_3P_4P_5 & \\
 & & & P_2P_3P_4P_6 & \\
 & & & P_2P_3P_5P_6 &
\end{array}
\qquad \beta = (\tfrac{18}{50}, \tfrac{14}{50}, \tfrac{14}{50}, \tfrac{2}{50}, \tfrac{2}{50}, 0)$$

$$2.9b.13 \quad P_1P_4P_5P_6 \qquad \beta = (\tfrac{19}{52}, \tfrac{13}{52}, \tfrac{13}{52}, \tfrac{3}{52}, \tfrac{3}{52}, \tfrac{1}{52})$$

2.9c.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_2P_3P_4$	$P_1P_3P_5P_6$	
			$P_1P_4P_5P_6$	
			$P_2P_3P_4P_5$	
			$P_2P_3P_4P_6$	

$\beta = (\frac{22}{50}, \frac{10}{50}, \frac{10}{50}, \frac{6}{50}, \frac{2}{50}, 0)$

2.10a.10	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_1P_4P_6$	$P_1P_3P_5P_6$	
		$P_1P_5P_6$	$P_1P_4P_5P_6$	

$\beta = (\frac{27}{46}, \frac{5}{46}, \frac{5}{46}, \frac{3}{46}, \frac{3}{46}, \frac{3}{46})$

2.10a.11 $P_2P_3P_4P_5$ $\beta = (\frac{26}{48}, \frac{6}{48}, \frac{6}{48}, \frac{4}{48}, \frac{4}{48}, \frac{2}{48})$

2.10a.12 $P_2P_3P_4P_5$
$P_2P_3P_4P_6$ $\beta = (\frac{25}{50}, \frac{7}{50}, \frac{7}{50}, \frac{5}{50}, \frac{3}{50}, \frac{3}{50})$

2.10a.13 $P_2P_3P_4P_5$
$P_2P_3P_4P_6$ $P_2P_3P_5P_6$ $\beta = (\frac{24}{52}, \frac{8}{52}, \frac{8}{52}, \frac{4}{52}, \frac{4}{52}, \frac{4}{52})$

2.10b.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_2P_3P_4$	$P_1P_3P_4P_6$	
		$P_2P_3P_5$	$P_1P_3P_5P_6$	
		$P_2P_3P_6$	$P_2P_3P_4P_5$	
			$P_2P_3P_4P_6$	
			$P_2P_3P_5P_6$	

$\beta = (\frac{17}{50}, \frac{15}{50}, \frac{15}{50}, \frac{1}{50}, \frac{1}{50}, \frac{1}{50})$

2.10b.13 $P_1P_4P_5P_6$ $\beta = (\frac{18}{52}, \frac{14}{52}, \frac{14}{52}, \frac{2}{52}, \frac{2}{52}, \frac{2}{52})$

2.10c.12	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_1P_4P_6$	$P_1P_3P_5P_6$	
		$P_2P_3P_4$	$P_1P_4P_5P_6$	
			$P_2P_3P_4P_5$	
			$P_2P_3P_4P_6$	

$\beta = (\frac{23}{50}, \frac{9}{50}, \frac{9}{50}, \frac{7}{50}, \frac{1}{50}, \frac{1}{50})$

2.10c.13 $P_2P_3P_5P_6$ $\beta = (\frac{22}{52}, \frac{10}{52}, \frac{10}{52}, \frac{6}{52}, \frac{2}{52}, \frac{2}{52})$

2.10d.13	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
		$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_2P_3P_4$	$P_1P_3P_5P_6$	
		$P_2P_3P_5$	$P_1P_4P_5P_6$	
			$P_2P_3P_4P_5$	
			$P_2P_3P_4P_6$	
			$P_2P_3P_5P_6$	

$$\beta = (\frac{20}{52}, \frac{12}{52}, \frac{12}{52}, \frac{4}{52}, \frac{4}{52}, 0)$$

3.9.10	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
	P_1P_4	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_1P_4P_6$	$P_1P_3P_5P_6$	
			$P_1P_4P_5P_6$	

$$\beta = (\frac{27}{44}, \frac{5}{44}, \frac{5}{44}, \frac{5}{44}, \frac{1}{44}, \frac{1}{44})$$

3.9.11 $P_2P_3P_4P_5$ $\beta = (\frac{26}{46}, \frac{6}{46}, \frac{6}{46}, \frac{6}{46}, \frac{2}{46}, 0)$

3.9.12 $P_2P_3P_4P_5$, $P_2P_3P_4P_6$ $\beta = (\frac{25}{48}, \frac{7}{48}, \frac{7}{48}, \frac{7}{48}, \frac{1}{48}, \frac{1}{48})$

3.10a.10	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$
	P_1P_4	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$
		$P_1P_3P_6$	$P_1P_3P_4P_5$	
		$P_1P_4P_5$	$P_1P_3P_4P_6$	
		$P_1P_4P_6$	$P_1P_3P_5P_6$	
		$P_1P_5P_6$	$P_1P_4P_5P_6$	

$$\beta = (\frac{28}{44}, \frac{4}{44}, \frac{4}{44}, \frac{4}{44}, \frac{2}{44}, \frac{2}{44})$$

3.10a.11 $P_2P_3P_4P_5$ $\beta = (\frac{27}{46}, \frac{5}{46}, \frac{5}{46}, \frac{5}{46}, \frac{3}{46}, \frac{1}{46})$

3.10a.12 $P_2P_3P_4P_5$, $P_2P_3P_4P_6$ $\beta = (\frac{26}{48}, \frac{6}{48}, \frac{6}{48}, \frac{6}{48}, \frac{2}{48}, \frac{2}{48})$

$3.10b.12$	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	P_1P_4	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{24}{48}, \frac{8}{48}, \frac{8}{48}, \frac{8}{48}, 0, 0)$
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_1P_4P_5$	$P_1P_3P_4P_6$		
		$P_1P_4P_6$	$P_1P_3P_5P_6$		
		$P_2P_3P_4$	$P_1P_4P_5P_6$		
			$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		

$3.10c.12$	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	P_2P_3	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
		$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{16}{48}, \frac{16}{48}, \frac{16}{48}, 0, 0, 0)$
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_2P_3P_4$	$P_1P_3P_4P_6$		
		$P_2P_3P_5$	$P_1P_3P_5P_6$		
		$P_2P_3P_6$	$P_2P_3P_4P_5$		
			$P_2P_3P_4P_6$		
			$P_2P_3P_5P_6$		

$4.10.10$	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	P_1P_4	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	P_1P_5	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
		$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	$\beta = (\frac{29}{42}, \frac{3}{42}, \frac{3}{42}, \frac{3}{42}, \frac{3}{42}, \frac{1}{42})$
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_1P_4P_5$	$P_1P_3P_4P_6$		
		$P_1P_4P_6$	$P_1P_3P_5P_6$		
		$P_1P_5P_6$	$P_1P_4P_5P_6$		

$4.10.11$ $P_2P_3P_4P_5$ $\beta = (\frac{28}{44}, \frac{4}{44}, \frac{4}{44}, \frac{4}{44}, \frac{4}{44}, 0)$

$5.10.10$	P_1P_2	$P_1P_2P_3$	$P_1P_2P_3P_4$	$P_1P_2P_3P_4P_5$	
	P_1P_3	$P_1P_2P_4$	$P_1P_2P_3P_5$	$P_1P_2P_3P_4P_6$	
	P_1P_4	$P_1P_2P_5$	$P_1P_2P_3P_6$	$P_1P_2P_3P_5P_6$	
	P_1P_5	$P_1P_2P_6$	$P_1P_2P_4P_5$	$P_1P_2P_4P_5P_6$	
	P_1P_6	$P_1P_3P_4$	$P_1P_2P_4P_6$	$P_1P_3P_4P_5P_6$	
		$P_1P_3P_5$	$P_1P_2P_5P_6$	$P_2P_3P_4P_5P_6$	$\beta = (\frac{30}{40}, \frac{2}{40}, \frac{2}{40}, \frac{2}{40}, \frac{2}{40}, \frac{2}{40})$
		$P_1P_3P_6$	$P_1P_3P_4P_5$		
		$P_1P_4P_5$	$P_1P_3P_4P_6$		
		$P_1P_4P_6$	$P_1P_3P_5P_6$		
		$P_1P_5P_6$	$P_1P_4P_5P_6$		

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