Math 461-A: Abstract Algebra I

Course Syllabus

Description: *A course in the theory of mathematical structures, i.e. groups, rings, and fields, along with substructures, quotient structures, and homomorphisms.*

Credit Hours: 3

Audience: Partially satisfies the theory elective requirement to complete a mathematics major.

Prerequisites: Math 266; prerequisite for MATH 462

Format: 3 lectures (50 min) per week

Textbook: *Contemporary Abstract Algebra 9th Edition* by Joseph A. Gallian. ISBN: 978-7-305-65796-0. (*Required)*

Technology: Proficiency in LaTeX typesetting is expected, and is required for the proper completion of homework and other take-home assignments. Course material and grades will be maintained on Blackboard (blackboard.valpo.edu).

Internet: Course material and grades will be maintained in Blackboard.

Course Goals

A. Students improve mathematical proof writing skills.
B. Students understand fundamental concepts of groups, rings, and homomorphisms.
C. Students improve problem solving skills.
D. Students can read and interpret technical mathematics writing.
E. Students prepare for success in future pure (abstract) mathematics classes.

Topical Objectives (with goals addressed). Preface: *Students will be able to …*

1. Define equivalence relation and equivalence class and determine, with complete justification, whether or not a given relation is an equivalence relation and, if so, identify equivalence classes. (A, C, E)
2. Perform modular arithmetic. (C, E)
3. Demonstrate familiarity with the definition of a group and be able to test a set with binary operation to determine if it is a group. (A, B, E)
4. Demonstrate familiarity with the common groups (e.g. S_n, Z_n, D_n, …). (B, E)
5. Compute the order of a group, the order of a subgroup, and the order of an element (B, C)
6. Identify subgroups of a given group. (A, B, C)
7. Apply Lagrange's theorem. (B, E)

8. Define normal subgroups and be able to prove that given subgroups are normal. (A, B)
9. Demonstrate familiarity with permutation groups and be able to decompose permutations into 2-cycles (A,C, D)
10. Define the concepts of homomorphism, isomorphism, and automorphism and prove whether a given function defines one of these. (A, B, D, E)
11. Prove the common properties of homomorphisms (e.g. that if K is a subgroup of G then $\phi(K)$ is a subgroup of ϕ (G)). (A, B, D)
12. State and apply the fundamental theorem of finite Abelian groups (B, C)
13. Give a definition of ring and cite a variety of common examples and non-examples (finite and infinite, polynomials, and matrices) (B, E)
14. Give the definition of field and cite a variety of common examples and non-examples (characteristic 0 and characteristic p, polynomials, matrices) (B, E)
15. Demonstrate familiarity with some of the applications of algebra to other fields, e.g. cryptography. (E)

General Objectives (with goals addressed). Preface: *Students will be able to …*

16. Carry over and apply knowledge from previous proof-based courses (A, D)
17. Use proper mathematical notation and vocabulary (D. E)
18. Write clear and detailed proofs to assigned theorems (A, D)
19. Write clear and detailed solutions to assigned computational problems. (C, E)