Engineering Economics

Cash Flow

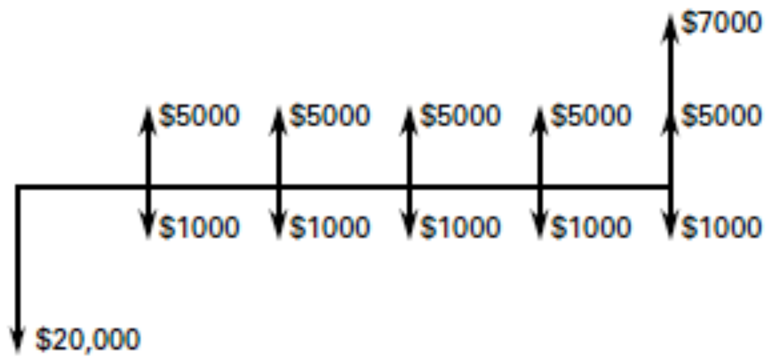
Cash flow is the sum of money recorded as receipts or disbursements in a project's financial records.

A cash flow diagram presents the flow of cash as arrows on a time line scaled to the magnitude of the cash flow, where expenses are down arrows and receipts are up arrows.

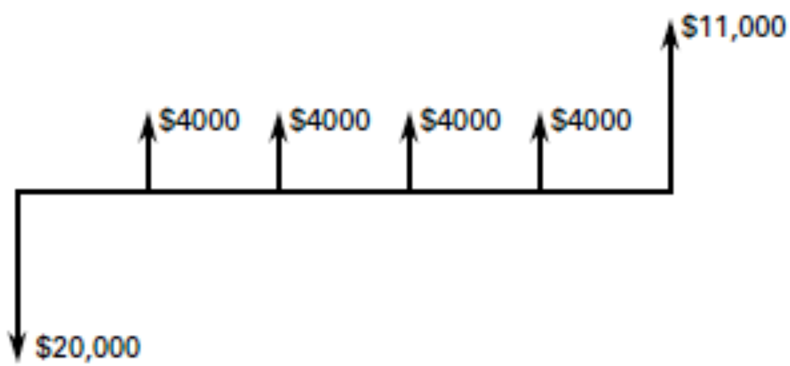
Year-end convention ~ expenses occurring during the year are assumed to occur at the end of the year.

Example (FEIM):
A mechanical device will cost $20,000 when purchased. Maintenance will cost $1000 per year. The device will generate revenues of $5000 per year for 5 years. The salvage value is $7000.

Figure 51.1 Cash Flow Diagrams



(a) cash flow diagram



(b) simplified cash flow diagram

Discount Factors and Equivalence

Present Worth (P): present amount at $t = 0$

Future Worth (F): equivalent future amount at $t = n$ of any present amount at $t = 0$

Annual Amount (A): uniform amount that repeats at the end of each year for n years

Uniform Gradient Amount (G): uniform gradient amount that repeats at the end of each year, starting at the end of the second year and stopping at the end of year n.

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Discount Factors and Equivalence

Table 51.1 Discount Factors for Discrete Compounding

NOTE: To save time, use the calculated factor table provided in the NCEES FE Handbook.

factor name	converts	symbol	formula
single payment compound amount	P to F	$(F/P, i\%, n)$	$(1+i)^n$
single payment present worth	F to P	$(P/F, i\%, n)$	$(1+i)^{-n}$
uniform series sinking fund	F to A	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n - 1}$
capital recovery	P to A	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n - 1}$
uniform series compound amount	A to F	$(F/A, i\%, n)$	$\frac{(1+i)^n - 1}{i}$
uniform series present worth	A to P	$(P/A, i\%, n)$	$\frac{(1+i)^n - 1}{i(1+i)^n}$
uniform gradient present worth	G to P	$(P/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}$
uniform gradient future worth	G to F	$(F/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}$
uniform gradient uniform series	G to A	$(A/G, i\%, n)$	$\frac{1}{i} - \frac{n}{(1+i)^n - 1}$

Discount Factors and Equivalence

Example (FEIM):
How much should be put in an investment with a 10% effective annual rate today to have \$10,000 in five years?

Using the formula in the factor conversion table,

$$P = F(1 + i)^{-n} = (\$10{,}000)(1 + 0.1)^{-5} = \$6209$$

Or using the factor table for 10%,

$$P = F(P/F, i\%, n) = (\$10{,}000)(0.6209) = \$6209$$

Discount Factors and Equivalence

Example (FEIM):

What factor will convert a gradient cash flow ending at $t = 8$ to a future value? The effective interest rate is 10%.

The F/G conversion is not given in the factor table. However, there are different ways to get the factor using the factors that are in the table. For example,

$$(F/G,i\%,8) = (P/G,10\%,8)(F/P,10\%,8)$$
$$= (16.0287)(2.1436)$$
$$= 34.3591$$

or

$$(F/G,i\%,8) = (F/A,10\%,8)(A/G,10\%,8)$$
$$= (11.4359)(3.0045)$$
$$= 34.3592$$

NOTE: The answers arrived at using the formula versus the factor table turn out to be slightly different. On economics problems, one should not worry about getting the exact answer.

Nonannual Compounding

Effective Annual Interest Rate
An interest rate that is compounded more than once in a year is converted from a compound nominal rate to an annual effective rate.

Effective Interest Rate Per Period

$$i = \frac{r}{m} \qquad 51.8$$

Effective Annual Interest Rate

$$\begin{aligned} i_e &= (1+i)^m - 1 \\ &= \left(1 + \frac{r}{m}\right)^m - 1 \qquad 51.9 \end{aligned}$$

Example (FEIM):
A savings and loan offers a 5.25% rate per annum compound daily over 365 days per year. What is the effective annual rate?

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.0525}{365}\right)^{365} - 1 = 0.0539$$

Discount Factors for Continuous Compounding

The formulas for continuous compounding are the same formulas in the factor conversion table with the limit taken as the number of periods, n, goes to infinity.

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Comparison of Alternatives

Present Worth

When alternatives do the same job and have the same lifetimes, compare them by converting each to its cash value today. The superior alternative will have the highest present worth.

Example (EIT8):

Example 13.11

Investment A costs \$10,000 today and pays back \$11,500 two years from now. Investment B costs \$8000 today and pays back \$4500 each year for two years. If an interest rate of 5% is used, which alternative is superior?

(solution)

The solution to this example is not difficult, but it will be postponed until methods of comparing alternatives have been covered.

Example 13.11 continued

(solution)

$$\begin{aligned} P(\mathrm{A}) &= -10{,}000 + 11{,}500(P/F, 5\%, 2) \\ &= -10{,}000 + (11{,}500)(0.9070) \\ &= \$431 \\ P(\mathrm{B}) &= -8000 + 4500(P/A, 5\%, 2) \\ &= -8000 + (4500)(1.8594) \\ &= \$367 \end{aligned}$$

Alternative A is superior and should be chosen.

Comparison of Alternatives

Capitalized Costs
Used for a project with infinite life that has repeating expenses every year.

Compare alternatives by calculating the capitalized costs (i.e., the amount of money needed to pay the start-up cost and to yield enough interest to pay the annual cost without touching the principal).

NOTE: The factor conversion for a project with no end is the limit of the *P*/*A* factor as the number of periods, *n*, goes to infinity.

$$\text{capitalized cost} = P = \frac{A}{i} \qquad \begin{bmatrix}\text{infinite}\\ \text{series}\end{bmatrix} \qquad 52.5$$

Comparison of Alternatives

Example (EIT8):

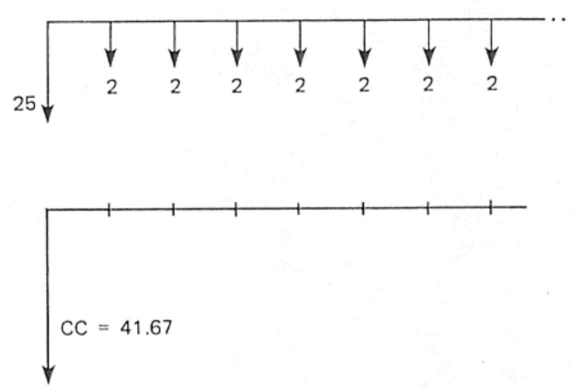
Example 13.12

What is the capitalized cost of a public works project that will cost $25,000,000 now and will require $2,000,000 in maintenance annually? The effective annual interest rate is 12%.

(solution)

Worked in millions of dollars, from Eq. 13.19, the capitalized cost is

$$\text{capitalized cost} = 25 + 2(P/A, 12\%, \infty)$$
$$= 25 + \frac{2}{0.12} = \$41.67$$



Comparison of Alternatives

Annual Cost
When alternatives do the same job but have different lives, compare the cost per year of each alternative.

The alternatives are assumed to be replaced at the end of their lives by identical alternatives. The initial costs are assumed to be borrowed at the start and repaid evenly during the life of the alternative.

Example (EIT8):

Example 13.13

Which of the following alternatives is superior over a 30-year period if the interest rate is 7%?

	alternative A	alternative B
type	brick	wood
life	30 years	10 years
initial cost	$1800	$450
maintenance	$5/year	$20/year

(solution)

$$\begin{aligned}\text{EUAC(A)} &= 1800(A/P, 7\%, 30) + 5\\ &= (1800)(0.0806) + 5\\ &= \$150\\ \text{EUAC(B)} &= 450(A/P, 7\%, 10) + 20\\ &= (450)(0.1424) + 20\\ &= \$84\end{aligned}$$

Comparison of Alternatives

Cost-Benefit Analysis
Project is considered acceptable if $B - C \geq 0$ or $B/C \geq 1$.

Example (FEIM):

The initial cost of a proposed project is \$40M, the capitalized perpetual annual cost is \$12M, the capitalized benefit is \$49M, and the residual value is \$0. Should the project be undertaken?

B = \$49M, C = \$40M + \$12M + \$0
$B - C$ = \$49M – \$52M = –\$3M < 0

The project should not be undertaken.

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Comparison of Alternatives

Rate of Return on an Investment (ROI)
The ROI must exceed the minimum attractive rate of return (MARR).

The rate of return is calculated by finding an interest rate that makes the present worth zero. Often this must be done by trial and error.

Depreciation

Straight Line Depreciation

The depreciation per year is the cost minus the salvage value divided by the years of life.

$$D_j = \frac{C - S_n}{n} \qquad 52.2$$

Depreciation

Accelerated Cost Recovery System (ACRS)

The depreciation per year is the cost times the ACRS factor (see the table in the NCEES Handbook). Salvage value is not considered.

$$D_j = C \times \text{factor} \qquad 52.3$$

Depreciation

Example (FEIM):

An asset is purchased that costs $9000. It has a 10-year life and a salvage value of $200. Find the straight-line depreciation and ACRS depreciation for 3 years.

$$\text{Straight-line depreciation/year} = \frac{\$9000 - \$200}{10} = \$880/\text{yr}$$

ACRS depreciation

First year ($9000)(0.1) = $ 900
Second year ($9000)(0.18) = $1620
Third year ($9000)(0.144) = $1296

Depreciation

Book Value

The assumed value of the asset after *j* years. The book value (BV_j) is the initial cost minus the sum of the depreciations out to the j^{th} year.

$$\begin{aligned} \mathrm{BV}_j &= \text{initial cost} - \text{accumulated depreciation} \\ &= C - \sum_{j=1}^{t} D_j \end{aligned} \qquad 52.4$$

Example (FEIM):

What is the book value of the asset in the previous example after 3 years using straight-line depreciation? Using ACRS depreciation?

Straight-line depreciation
$9000 – (3)($800) = $6360

ACRS depreciation
$9000 – $900 – $1620 – $1296 = $5184

Tax Considerations

Expenses and depreciation are deductible, revenues are taxed.

Example (EIT8):

Example 13.19

A corporation that pays 53% of its profit in income taxes invests \$10,000 in an asset that will produce \$3000 annual revenue for eight years. If the annual expenses are \$700, salvage after eight years is \$500, and 9% interest is used, what is the after-tax present worth? Disregard depreciation.

(solution)

$$\begin{aligned} P &= -10{,}000 + 3000(P/A, 9\%, 8)(1 - 0.53) \\ &\quad - 700(P/A, 9\%, 8)(1 - 0.53) \\ &\quad + 500(P/F, 9\%, 8) \\ &= -10{,}000 + (3000)(5.5348)(0.47) \\ &\quad - (700)(5.5348)(0.47) \\ &\quad + (500)(0.5019) \\ &= -\$3766 \end{aligned}$$

Tax Considerations

Tax Credit
A one-time benefit from a purchase that is subtracted from income taxes.

Example (EIT8):

Example 13.20

One year, a company makes a $5000 investment in a historic building. The investment is not depreciable, but it does qualify for a one-time 20% tax credit. In that same year, revenue is $45,000 and expenses (exclusive of the $5000 investment) are $25,000. The company pays a total of 53% in income taxes. What is the after-tax present worth of this year's activities if the company's interest rate for investment is 10%?

(solution)

The tax credit is

$$\text{TC} = (0.20)(5000) = \$1000$$

This tax credit is assumed to be received at the end of the year. The after-tax present worth is

$$\begin{aligned} P &= -5000 + (45{,}000 - 25{,}000)(1 - 0.53)(P/F, 10\%, 1) \\ &\quad + 1000(P/F, 10\%, 1) \\ &= -5000 + (20{,}000)(0.47)(0.9091) \\ &\quad + (1000)(0.9091) \\ &= \$4455 \end{aligned}$$

Tax Considerations

Gain or loss on the sale of an asset:

If an asset has been depreciated and then is sold for more than the book value, the difference is taxed.

Bonds

Bond value is the present worth of payments over the life of the bond.

Bond yield is the equivalent interest rate of the bond compared to the bond cost.

Example (EIT8):

Example 13.24

What is the maximum amount an investor should pay for a 25-year bond with a \$20,000 face value and 8% coupon rate (interest only paid semiannually)? The bond will be kept to maturity. The investor's effective annual interest rate for economic decisions is 10%.

(solution)

For this problem, take the compounding period to be six months. Then, there are 50 compounding periods. Since 8% is a nominal rate, the effective bond rate per period is calculated from Eq. 13.53 as $\phi_{\text{bond}} = r/k = 8\%/2 = 4\%$.

The bond payment received semiannually is

$$(0.04)(\$20,000) = \$800$$

10% is the investor's effective rate per year, so Eq. 13.54 is again used to calculate the effective analysis rate per period.

$$0.10 = (1+\phi)^2 - 1$$
$$\phi = 0.04881\ (4.88\%)$$

The maximum amount that the investor should be willing to pay is the present worth of the investment.

$$P = 800(P/A, 4.88\%, 50) + 20,000(P/F, 4.88\%, 50)$$

Table 13.1 can be used to calculate the factors:

$$(P/A, 4.88\%, 50) = \frac{(1.0488)^{50} - 1}{(0.0488)(1.0488)^{50}} = 18.600$$

$$(P/F, 4.88\%, 50) = \frac{1}{(1.0488)^{50}} = 0.09233$$

Then, the present worth is

$$\begin{aligned} P &= (\$800)(18.600) + (\$20,000)(0.09233) \\ &= \$16,727 \end{aligned}$$

Break-Even Analysis

Calculating when revenue is equal to cost, or when one alternative is equal to another if both depend on some variable.

Example (FEIM):
How many kilometers must a car be driven per year for leasing and buying to cost the same? Use 10% interest and year-end cost.

Leasing: \$0.15 per kilometer

Buying: \$5000 purchase cost, 3-year life, salvage \$1200,
\$0.04 per kilometer for gas and oil, \$500 per year for insurance

EUAC (leasing) = $\$0.15x$, where x is kilometers driven

$$
\begin{aligned}
\text{EUAC (buying)} &= \$0.04x + \$500 + (\$5\text{k})(A/P,10\%,3) - (\$1.2\text{k})(A/F,10\%,3) \\
&= \$0.04\text{x} + \$500 + (\$5\text{k})(0.4021) - (\$1.2\text{k})(0.3021) \\
&= \$0.04x + \$2148
\end{aligned}
$$

Setting EUAC (leasing) = EUAC (buying) and solving for x

$$
\begin{aligned}
\$0.15x &= \$0.04x + \$2148 \\
x &= 19{,}527 \text{ km must be driven to break even}
\end{aligned}
$$

Inflation

Inflation-Adjusted Interest Rate

$$d = i + f + if \qquad 52.6$$

Additional Examples

Example 1 (FEIM):

What is the uninflated present worth of $2000 in 2 years if the average inflation rate is 6% and i is 10%?

$d = i + f + if = 0.06 + 0.10 + (0.06)(0.10) = 0.166$

$P = (\$2000)(P/F,16.6\%,2) = (\$2000)(1 + d)^{-n}$

$= (\$2000)(1 + 0.166)^{-2} = \$1471$

Additional Examples

Example 2 (FEIM):

It costs \$75 per year to maintain a cemetery plot. If the interest rate is 6.0%, how much must be set aside to pay for maintenance on each plot without touching the principal?

(A) \$1150
(B) \$1200
(C) \$1250
(D) \$1300

$P = (\$75)(P/A,6\%,\infty) = (\$75)(1/0.06) = \$1250$

Therefore, (C) is correct.

Additional Examples

Example 3 (FEIM):

It costs \$1000 for hand tools and \$1.50 labor per unit to manufacture a product. Another alternative is to manufacture the product by an automated process that costs \$15,000, with a \$0.50 per-unit cost. With an annual production rate of 5000 units, how long will it take to reach the break-even point?

(A) 2.0 yr
(B) 2.8 yr
(C) 3.6 yr
(D) never

Cumulative cost (hand tools) = \$1000 + \$1.50x, where x is the number of units.
Cumulative cost (automated) = \$15,000 + \$0.50x
Set cumulative costs equal and solve for x.

$$\$1000 + \$1.50x = \$15,000 + \$0.50x$$
$$\$1x = \$14,000$$
$$x = 14,000 \text{ units}$$
$$t_{\text{break-even}} = x/\text{production rate} = 14,000/5000 = 2.8 \text{ yr}$$

Therefore, (B) is correct.

Additional Examples

Example 4 (FEIM):

A loan of $10,000 is made today at an interest rate of 15%, and the first payment of $3000 is made 4 years later. The amount that is still due on the loan after the first payment is most nearly

(A) $7000
(B) $8050
(C) $8500
(D) $14,500

$$\begin{aligned}\text{loan due} &= (\$10\text{k})(F/P,15\%,4) - \$3000\\ &= (\$10\text{k})(1 + 0.15)^4 - \$3000\\ &= (\$10\text{k})(1.7490) - \$3000\\ &= \$14{,}490 \quad (\$14{,}500)\end{aligned}$$

Therefore, (D) is correct.

Additional Examples

Example 5 (FEIM):

A machine is purchased for $1000 and has a useful life of 12 years. At the end of 12 years, the salvage value is $130. By straight-line depreciation, what is the book value of the machine at the end of 8 years?

(A) $290
(B) $330
(C) $420
(D) $580

BV = \$1000 – (\$1000 – \$130)(8/12) = \$1000 – \$580 = \$420

Therefore, (C) is correct.

Additional Examples

Example 6 (FEIM):

The maintenance cost for an investment is \$2000 per year for the first 10 years and \$1000 per year thereafter. The investment has infinite life. With a 10% interest rate, the present worth of the annual disbursement is most nearly

(A) \$10,000
(B) \$16,000
(C) \$20,000
(D) \$24,000

The costs or benefits for a cash flow that repeat should be broken into different benefits and costs that all start or finish at the time of interest. Take the \$2000 cost that repeats for 10 years and break it into two \$1000 costs to have one \$1000 cost that goes on infinitely and one \$1000 cost that goes on for 10 years.

$$P = (\$1000)(P/A,10\%,10) + (\$1000)(P/A,10\%,\infty)$$
$$= (\$1000)(6.1446) + (\$1000)(1/0.10)$$
$$= \$6144.6 + \$10{,}000 = \$16{,}144.6 \quad (\$16{,}000)$$

Therefore, (B) is correct.

Additional Examples

Example 7 (FEIM):

With an interest rate of 8% compounded semiannually, the value of a $1000 investment after 5 years is most nearly

(A) $1400
(B) $1470
(C) $1480
(D) $1800

$$i_e = (1 + r/m)^m - 1 = (1 + 0.08/2)^2 - 1 = 0.0816$$
$$F = (\$1000)(F/P,8.16\%,5) = (\$1000)(1 + 0.0816)^5$$
$$= (\$1000)(1.480) = \$1480$$

Therefore, (C) is correct.

Additional Examples

Example 8 (FEIM):
The following data applies for example problems 8.1 through 8.3. A company is considering the purchase of either machine A or machine B.

	machine A	machine B
initial cost	$80,000	$100,000
estimated life	20 years	25 years
salvage value	$20,000	$25,000
other costs	$18,000 per year	$15,000 per year for the first 15 years
		$20,000 per year for the next 10 years

Example 8.1 (FEIM):
The interest rate is 10%, and all cash flows may be treated as end-of-year cash flows. Assume that equivalent annual cost is the value of the constant annuity equal to the total cost of a project. The equivalent annual cost of machine B is most nearly

(A) $21,000
(B) $21,500
(C) $23,000
(D) $26,500

Additional Examples

The \$15,000 cost for 15 years and the \$20,000 cost for the next 10 years can be broken into a \$20,000 cost for the full 25 years and a \$5000 benefit that is present for the first 15 years.

The present worth of \$20k for 25 years is

$P(\$20,25) = (\$20\text{k})(P/A,10\%,25) = (\$20\text{k})(9.0770) = \$181.54\text{k}$

The present worth of \$5k for 15 years is

$P(\$5,15) = (\$5\text{k})(P/A,10\%,15) = (\$5\text{k})(7.6061) = 38.03\text{k}$

$P_{\text{other costs}} = \$181.54\text{ k} + \$38.03\text{k} = \$143.51\text{k}$

$A_{\text{other costs}} = P_{\text{other costs}}(A/P, 10\%, 25) = (\$143.51\text{k})(0.1102) = \$15{,}815$

$$\begin{aligned}\text{EUAC} &= (\$100\text{k})(A/P,10\%,25) - (\$25\text{k})(A/F,10\%,25) + \$15{,}815 \\ &= (\$100\text{k})(0.1102) - (\$25\text{k})(0.0102) + \$15{,}815 \\ &= \$11{,}020 - \$255 + \$15{,}815 = \$26{,}610 \quad (\$26{,}500)\end{aligned}$$

Therefore, (D) is correct.

Additional Examples

Example 8.2 (FEIM):

If funds equal to the present worth of the cost of purchasing and using machine A over 20 years were invested at 10% per annum, the value of the investment at the end of 20 years would be most nearly:

(A) $548,000
(B) $676,000
(C) $880,000
(D) $1,550,000

$$
\begin{aligned}
P(A) &= \$80\text{k} + (\$20\text{k})(P/F,10\%,20) - (\$18\text{k})(P/A,10\%,20) \\
&= \$80\text{k} + (\$20\text{k})(0.1486) - (\$18\text{k})(8.5136) \\
&= \$80\text{k} + \$2.972\text{k} - \$153.245\text{k} \\
&= \$230{,}273
\end{aligned}
$$

$$
\begin{aligned}
F(A,10\%,20) &= (\$230{,}273)(F/P,10\%,20) = (\$230{,}273)(6.7275) \\
&= \$1{,}549{,}162 \ \ (\$1{,}550{,}000)
\end{aligned}
$$

Therefore, (D) is correct.

Additional Examples

Example 8.3 (FEIM):

How much money would have to be placed in a sinking fund each year to replace machine B at the end of 25 years if the fund yields 10% annual compound interest and if the first cost of the machine is assumed to increase at a 6% annual compound rate? (Assume the salvage value does not change.)

(A) \$2030
(B) \$2510
(C) \$2540
(D) \$4110

$F = P(F/P,6\%,25) - \text{salvage value} = (\$100{,}000)(4.2919) - \$25{,}000$
$\quad = \$404{,}190$
$A = F(A/F,10\%,25) = (\$404{,}190)(0.0102)$
$\quad = \$4123 \;\; (\$4110)$

Therefore, (D) is correct.