Fluid Mechanics

Definitions

Fluids

- Substances in either the liquid or gas phase
- Cannot support shear

Density

- Mass per unit volume

Specific Volume

\[ v = \frac{1}{\rho} \]

Specific Weight

\[ \gamma = \lim_{\Delta V \to 0} \left( \frac{g \Delta m}{\Delta V} \right) = \rho g \]

Specific Gravity

\[ SG = \frac{\rho}{\rho_{\text{water}}} = \frac{\gamma}{\gamma_{\text{water}}} \]
Example (FEIM):

Determine the specific gravity of carbon dioxide gas (molecular weight = 44) at 66°C and 138 kPa compared to STP air.

\[
R_{\text{carbon dioxide}} = \frac{8314 \, \text{J}}{44 \, \text{kg/mol}} = 189 \, \text{J/kg} \cdot \text{K}
\]

\[
R_{\text{air}} = \frac{8314 \, \text{J}}{29 \, \text{kg/mol}} = 287 \, \text{J/kg} \cdot \text{K}
\]

\[
SG = \frac{\rho}{\rho_{\text{STP}}} = \frac{PR_{\text{air}}T_{\text{STP}}}{R_{\text{CO}_2}T_{\text{PSTP}}} = \left( \frac{1.38 \times 10^5 \, \text{Pa}}{189 \, \text{J/kg} \cdot \text{K}(66^\circ \text{C} + 273.16)} \right) \left( \frac{287 \, \text{J/kg} \cdot \text{K}(273.16)}{1.013 \times 10^5 \, \text{Pa}} \right) = 1.67
\]
Fluid Mechanics

Definitions

Shear Stress

- Normal Component: $\tau_n = p$

- Tangential Component
  - For a Newtonian fluid: $\tau_t = \mu \frac{dv}{dy}$
  - For a pseudoplastic or dilatant fluid: $\tau_t = K \left( \frac{dv}{dy} \right)^n$
Fluid Mechanics

Definitions

Absolute Viscosity
  • Ratio of shear stress to rate of shear deformation

Surface Tension

\[ \sigma = \frac{F}{L} \]  \hspace{1cm} 22.14

Capillary Rise

\[ h = \frac{4\sigma \cos \beta}{\rho d_{\text{tube}} g} \]  \hspace{1cm} \text{[SI]} \hspace{1cm} 22.17a
Fluid Mechanics

Definitions

Example (FEIM):
Find the height to which ethyl alcohol will rise in a glass capillary tube 0.127 mm in diameter.

Density is 790 kg/m$^3$, $\sigma = 0.0227$ N/m, and $\beta = 0^\circ$.

\[
h = \frac{4\sigma \cos \beta}{\gamma d} = \frac{(4)(0.0227 \text{ kg/s}^2)(1.0)}{(790 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.127 \times 10^{-3} \text{ m})} = 0.00923 \text{ m}
\]
Fluid Mechanics

Fluid Statics

Gage and Absolute Pressure
\[ p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atmospheric}} \]

Hydrostatic Pressure
\[ p = \gamma h + \rho gh \]
\[ p_2 - p_1 = -\gamma (z_2 - z_1) \]

Example (FEIM):
In which fluid is 700 kPa first achieved?

(A) ethyl alcohol
(B) oil
(C) water
(D) glycerin

\begin{align*}
\text{60 m} & \quad \text{ethyl alcohol} & 7.586 \text{ kPa/m} \\
\text{10 m} & \quad \text{oil} & 8.825 \text{ kPa/m} \\
\text{5 m} & \quad \text{water} & 9.604 \text{ kPa/m} \\
\text{5 m} & \quad \text{glycerin} & 12.125 \text{ kPa/m} \\
\end{align*}

\[ p_0 = 90 \text{ kPa} \]
Therefore, (D) is correct.

\[ p_0 = 90 \text{ kPa} \]
\[ p_1 = p_0 + \gamma_1 h_1 = 90 \text{ kPa} + \left( 7.586 \frac{kPa}{m} \right)(60 \text{ m}) = 545.16 \text{ kPa} \]
\[ p_2 = p_1 + \gamma_2 h_2 = 545.16 \text{ kPa} + \left( 8.825 \frac{kPa}{m} \right)(10 \text{ m}) = 633.41 \text{ kPa} \]
\[ p_3 = p_2 + \gamma_3 h_3 = 633.41 \text{ kPa} + \left( 9.604 \frac{kPa}{m} \right)(5 \text{ m}) = 681.43 \text{ kPa} \]
\[ p_4 = p_3 + \gamma_4 h_4 = 681.43 \text{ kPa} + \left( 12.125 \frac{kPa}{m} \right)(5 \text{ m}) = 742 \text{ kPa} \]
Manometers

$p_o - p_a = \gamma_2 h_2 - \gamma_1 h_1$  

[U.S.] 23.4b
Example (FEIM):
The pressure at the bottom of a tank of water is measured with a mercury manometer. The height of the water is 3.0 m and the height of the mercury is 0.43 m. What is the gage pressure at the bottom of the tank?

From the table in the NCEES Handbook,

\[ \rho_{\text{mercury}} = 13560 \text{ kg/m}^3 \]
\[ \rho_{\text{water}} = 997 \text{ kg/m}^3 \]

\[ \Delta p = g \left( \rho_2 h_2 - \rho_1 h_1 \right) \]
\[ = \left( 9.81 \text{ m/s}^2 \right) \left( 13560 \text{ kg/m}^3 \right) (0.43 \text{ m}) - \left( 997 \text{ kg/m}^3 \right) (3.0 \text{ m}) \]
\[ = 27858 \text{ Pa} \]
Barometer

Atmospheric Pressure

\[ p_a - p_v = \rho gh \]  

[SI]  

23.7a
Fluid Mechanics

Fluid Statics

Forces on Submerged Surfaces

\[ R = pA \]

\[ \bar{p} = \frac{1}{2} \rho g (h_1 + h_2) \]  \[23.8\]  \[23.10a\]

Example (FEIM):
The tank shown is filled with water. Find the force on 1 m width of the inclined portion.

The average pressure on the inclined section is:

\[ p_{\text{ave}} = \left( \frac{1}{2} \right) \left( 997 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) (3 \text{ m} + 5 \text{ m}) \]

\[ = 39122 \text{ Pa} \]

The resultant force is

\[ R = p_{\text{ave}} A = (39122 \text{ Pa})(2.31 \text{ m})(1 \text{ m}) \]

\[ = 90372 \text{ N} \]
Center of Pressure

If the surface is open to the atmosphere, then \( p_0 = 0 \) and

\[
y^* = \frac{\rho g I_{yz} \sin \alpha}{p_c A} \quad [\text{SI}] \tag{23.17a}
\]

\[
z^* = \frac{\rho g I_{yy} \sin \alpha}{p_c A} \quad [\text{SI}] \tag{23.18a}
\]

\[
p_c = \bar{p} = \rho g z_c \sin \alpha \quad [\text{SI}] \tag{23.19a}
\]

\[
y_{cp} - y_c = y^* = \frac{I_{yz}}{z_c A} \tag{23.20}
\]

\[
z_{cp} - z_c = z^* = \frac{I_{yy}}{z_c A} \tag{23.21}
\]
Example 1 (FEIM):
The tank shown is filled with water. At what depth does the resultant force act?

The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

\[ A = bh \]
\[ I_{yc} = \frac{b^3h}{12} \]
\[ Z_c = \frac{4 \text{ m}}{\sin 60^\circ} = 4.618 \text{ m} \]
Fluid Mechanics

Fluid Statics

Using the moment of inertia for a rectangle given in the NCEES Handbook,

\[ z^* = \frac{I_{yc}}{AZ_c} = \frac{b^3h}{12bhZ_c} = \frac{b^2}{12Z_c} \]

\[ = \frac{(2.31 \text{ m})^2}{(12)(4.618 \text{ m})} = 0.0963 \text{ m} \]

\[ R_{\text{depth}} = (Z_c + z^*) \sin 60^\circ = (4.618 \text{ m} + 0.0963 \text{ m}) \sin 60^\circ = 4.08 \text{ m} \]
Fluid Mechanics

Fluid Statics

Example 2 (FEIM):
The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m$^3$. The magnitude of the force $F$ per meter of width to keep the gate closed is most nearly

\[ p_{ave} = \rho g z_{ave} \left(1600 \text{ kg/m}^3 \right) \left(9.81 \text{ m/s}^2 \right) \left(\frac{1}{2}\right)(3 \text{ m}) \]
\[ = 23544 \text{ Pa} \]
\[ R \frac{w}{w} = p_{ave} h = (23544 \text{ Pa})(3 \text{ m}) = 70662 \text{ N/m} \]
\[ F + F_h = R \]
\[ R \]

$R$ is one-third from the bottom (centroid of a triangle from the NCEES Handbook).

Taking the moments about $R$,
\[ 2F = F_h \]
\[ F \frac{w}{w} = \left(\frac{1}{3}\right) \frac{R}{w} = \frac{70,667 \text{ N}}{3} = 23.6 \text{ kN/m} \]

Therefore, (B) is correct.

(A) 0 kN/m
(B) 24 kN/m
(C) 71 kN/m
(D) 370 kN/m
Archimedes’ Principle and Buoyancy

• The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.

• A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

\[ F_{\text{buoyant}} = \gamma_{\text{water}} V_{\text{displaced}} \]
Hydraulic Radius for Pipes

\[ R_H = \frac{\text{area in flow}}{\text{wetted perimeter}} \]

Example (FEIM):
A pipe has diameter of 6 m and carries water to a depth of 2 m. What is the hydraulic radius?

\( r = 3 \text{ m} \)
\( d = 2 \text{ m} \)
\( \phi = (2 \text{ m})(\arccos((r - d) / r)) = (2 \text{ m})(\arccos \frac{1}{3}) = 2.46 \text{ radians} \)

(Careful! Degrees are very wrong here.)

\( s = r\phi = (3 \text{ m})(2.46 \text{ radians}) = 7.38 \text{ m} \)
\( A = \frac{1}{2}(r^2(\phi - \sin\phi)) = \frac{1}{2}((3 \text{ m})^2(2.46 \text{ radians} - \sin2.46)) = 8.235 \text{ m}^2 \)

\[ R_H = \frac{A}{s} = \frac{8.235 \text{ m}^2}{7.38 \text{ m}} = 1.12 \text{ m} \]
Fluid Mechanics

Fluid Dynamics

Continuity Equation

\[ \dot{m} = \rho A \dot{v} = \rho Q \]

\[ \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \]

If the fluid is incompressible, then \( \rho_1 = \rho_2 \).

\[ Q = A_1 v_1 = A_2 v_2 \]
The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. The speed in the 130 mm pipe is most nearly

(A) 1 m/s  
(B) 2 m/s  
(C) 4 m/s  
(D) 16 m/s

\[ A_1 v_1 = A_2 v_2 \]
\[ A_1 = 4A_2 \]

so \[ v_2 = 4v_1 = \left(4 \left(4 \, \text{m/s}\right)\right) = 16 \, \text{m/s} \]

Therefore, (D) is correct.
Bernoulli Equation

\[
\frac{p_1}{\gamma_1} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma_2} + \frac{v_2^2}{2g} + z_2 \quad \text{[U.S.]} \quad 24.11b
\]

- In the form of energy per unit mass:

\[
\frac{p_1}{\rho_1} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho_2} + \frac{v_2^2}{2} + gz_2
\]
Example (FEIM):
A pipe draws water from a reservoir and discharges it freely 30 m below the surface. The flow is frictionless. What is the total specific energy at an elevation of 15 m below the surface? What is the velocity at the discharge?
Let the discharge level be defined as $z = 0$, so the energy is entirely potential energy at the surface.

$$E_{\text{surface}} = z_{\text{surface}} \cdot g = (30 \text{ m}) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) = 294.3 \text{ J/kg}$$

(Note that $\text{m}^2/\text{s}^2$ is equivalent to J/kg.)

The specific energy must be the same 15 m below the surface as at the surface.

$$E_{15 \text{ m}} = E_{\text{surface}} = 294.3 \text{ J/kg}$$

The energy at discharge is entirely kinetic, so

$$E_{\text{discharge}} = 0 + 0 + \frac{1}{2} v^2$$

$$v = \sqrt{(2) \left( 294.3 \frac{\text{J}}{\text{kg}} \right)} = 24.3 \text{ m/s}$$
Flow of a Real Fluid

- Bernoulli equation + head loss due to friction

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f \quad \text{[U.S.]} \quad 24.12b
\]

\[
h_f = \frac{p_1 - p_2}{\gamma} \quad \text{[U.S.]} \quad 24.13b
\]

\((h_f\) is the head loss due to friction)
Fluid Flow Distribution

If the flow is laminar (no turbulence) and the pipe is circular, then the velocity distribution is:

\[ v_r = v_{\text{max}} \left( 1 - \left( \frac{r}{R} \right)^2 \right) \]

\[ r = \text{the distance from the center of the pipe} \]
\[ v = \text{the velocity at } r \]
\[ R = \text{the radius of the pipe} \]
\[ v_{\text{max}} = \text{the velocity at the center of the pipe} \]
Reynolds Number
For a Newtonian fluid:

\[
Re = \frac{vD\rho}{\mu} \quad [\text{SI}] \quad 24.14a
\]

\[
Re = \frac{vD}{\nu} \quad 24.15
\]

\[D = \text{hydraulic diameter} = 4R_H\]
\[\nu = \text{kinematic viscosity}\]
\[\mu = \text{dynamic viscosity}\]

For a pseudoplastic or dilatant fluid:

\[
Re' = \frac{v^{2-n}D^n\rho}{K \left(\frac{3n+1}{4n}\right)^n 8^{n-1}} \quad 24.16
\]
Example (FEIM):
What is the Reynolds number for water flowing through an open channel 2 m wide when the flow is 1 m deep? The flow rate is 800 L/s. The kinematic viscosity is $1.23 \times 10^{-6}$ m$^2$/s.

\[
D = 4R_h = 4 \frac{A}{p} = \frac{(4)(1 \text{ m})(2 \text{ m})}{2 \text{ m} + 1 \text{ m} + 1 \text{ m}} = 2 \text{ m}
\]

\[
v = \frac{Q}{A} = \frac{800 \text{ L}}{2 \text{ m}^2} = 0.4 \text{ m/s}
\]

\[
Re = \frac{vD}{\nu} = \frac{\left(0.4 \text{ m/s}\right)(2 \text{ m})}{1.23 \times 10^{-6} \text{ m}^2/\text{s}} = 6.5 \times 10^5
\]
Hydraulic Gradient

- The decrease in pressure head per unit length of pipe

\[ m = \rho A v = \rho Q \]  

24.2

*Figure 24.2  Hydraulic Grade Line in a Horizontal Pipe*
Fluid Mechanics

Head Loss in Conduits and Pipes

Darcy Equation

- calculates friction head loss

\[ h_f = \frac{fLv^2}{2Dg} \]

Moody (Stanton) Diagram:

Fluid Mechanics

Head Loss in Conduits and Pipes

Minor Losses in Fittings, Contractions, and Expansions
• Bernoulli equation + loss due to fittings in the line and contractions or expansions in the flow area

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}}
\]

[U.S.] \hspace{1cm} 24.30b

\[
h_{L,\text{fitting}} = C \left( \frac{v^2}{2g} \right)
\]

24.31

Entrance and Exit Losses
• When entering or exiting a pipe, there will be pressure head loss described by the following loss coefficients:

- sharp exit \hspace{1cm} C = 1.0
- protruding pipe exit \hspace{1cm} C = 0.8
- sharp entrance \hspace{1cm} C = 0.5
- rounded entrance \hspace{1cm} C = 0.1
Fluid Mechanics

Pump Power Equation

\[ P = W = \frac{Q \gamma h}{\eta} = \frac{Q \rho gh}{\eta} = \frac{mg h}{\eta} \quad 25.1 \]
Fluid Mechanics

Impulse-Momentum Principle

\[ \sum F = Q_2 \rho_2 v_2 - Q_1 \rho_1 v_1 \] [SI] \quad 24.38a

Pipe Bends, Enlargements, and Contractions

\[-F_x = p_2 A_2 \cos \alpha - p_1 A_1 \]
\[ + Q \rho (v_2 \cos \alpha - v_1) \] [SI] \quad 24.39a

\[ F_y = (p_2 A_2 + Q \rho v_2) \sin \alpha + m_{\text{fluid}} g \] [SI] \quad 24.40a
Example (FEIM):
Water at 15.5°C, 275 kPa, and 997 kg/m^3 enters a 0.3 m × 0.2 m reducing elbow at 3 m/s and is turned through 30°. The elevation of the water is increased by 1 m. What is the resultant force exerted on the water by the elbow? Ignore the weight of the water.

\[ r_1 = \frac{0.3 \text{ m}}{2} = 0.15 \text{ m} \]
\[ r_2 = \frac{0.2 \text{ m}}{2} = 0.10 \text{ m} \]
\[ A_1 = \pi r_1^2 = \pi (0.15 \text{ m})^2 = 0.0707 \text{ m}^2 \]
\[ A_2 = \pi r_2^2 = \pi (0.10 \text{ m})^2 = 0.0314 \text{ m}^2 \]

By the continuity equation:
\[ v_2 = \frac{v_1 A_1}{A_2} = \frac{\left(3 \frac{\text{m}}{\text{s}}\right)(0.0707 \text{ m}^2)}{0.0314 \text{ m}^2} = 6.75 \text{ m/s} \]
Fluid Mechanics

Impulse-Momentum Principle

Use the Bernoulli equation to calculate \( p_2 \):

\[
p_2 = \rho \left( -\frac{v_2^2}{2} + \frac{p_1}{\rho} + \frac{v_1^2}{2} + g(z_1 - z_2) \right)
\]

\[
= \left( 997 \frac{\text{kg}}{\text{m}^3} \right) \left( -\frac{\left(6.75 \frac{\text{m}}{\text{s}}\right)^2}{2} + \frac{275000 \text{ Pa}}{997 \frac{\text{kg}}{\text{m}^3}} + \frac{\left(3 \frac{\text{m}}{\text{s}}\right)^2}{2} + \left(9.8 \frac{\text{m}}{\text{s}^2}\right)(0 \text{ m} - 1 \text{ m}) \right)
\]

\[
= 247000 \text{ Pa} \quad (247 \text{ kPa})
\]

\( Q = vA \)

\( F_x = -Q\rho(v_2 \cos \alpha - v_1) + P_1A_1 + P_2A_2 \cos \alpha \)

\[
= -(3)(0.0707) \left( 997 \frac{\text{kg}}{\text{m}^3} \right) \left( 6.75 \frac{\text{m}}{\text{s}} \cos 30^\circ - 3 \frac{\text{m}}{\text{s}} \right) + (275 \times 10^3 \text{ Pa})(0.0707)
\]

\[
+ (247 \times 10^3 \text{ Pa})(0.0314 \text{ m}^2)\cos 30^\circ
\]

\[
= 256 \times 10^4 \text{ N}
\]
Fluid Mechanics
Impulse-Momentum Principle

\[ F_y = Qp(v_2 \sin \alpha - 0) + P_2 A_2 \sin \alpha \]

\[ \begin{align*}
&= (3)(0.0707) \left( 997 \frac{\text{kg}}{\text{m}^3} \right) \left( 6.75 \frac{\text{m}}{\text{s}} \right) \sin 30^\circ \\
&\quad + (247 \times 10^3 \text{ Pa})(0.0314 \text{ m}^2) \sin 30^\circ \\
&= 4592 \times 10^4 \text{ N}
\end{align*} \]

\[ R = \sqrt{F_x^2 + F_y^2} = \sqrt{(25600 \text{ kN})^2 + (4592 \text{ kN})^2} = 26008 \text{ kN} \]
Fluid Mechanics

Impulse-Momentum Principle

Initial Jet Velocity: \[ v = \sqrt{2gh} \]  \hspace{1cm} 24.41

Jet Propulsion:
\[
F = \dot{m}(v_2 - v_1) \\
= \dot{m}(v_2 - 0) \\
= Q\rho v_2 \\
= v_2 A_2 \rho v_2 \\
= A_2 \rho v_2^2 \\
= A_2 \rho \left( \sqrt{2gh} \right)^2 \\
= 2g \rho h A_2 \\
= 2\gamma h A_2 \]  \hspace{1cm} 24.42
Fixed Blades

**Figure 24.9 Open Jet on a Stationary Blade**

\[ -F_x = Q \rho (v_2 \cos \alpha - v_1) \quad [\text{SI}] \quad 24.43a \]

\[ F_y = Q \rho v_2 \sin \alpha \quad [\text{SI}] \quad 24.44a \]
Moving Blades

\[ F_x = -Q\rho(v_1 - v)(1 - \cos \alpha) \quad \text{[SI]} \quad 24.45a \]
\[ F_y = Q\rho(v_1 - v) \sin \alpha \quad \text{[SI]} \quad 24.46a \]
**Fluid Mechanics**

**Impulse-Momentum Principle**

**Impulse Turbine**

The maximum power possible is the kinetic energy in the flow.

\[ P = Q\rho(v_1 - v)(1 - \cos \alpha)v \quad [\text{SI}] \quad 24.47a \]

\[ P_{\text{max}} = \frac{Q\rho v_1^2}{2} \quad [\text{SI}] \quad 24.49a \]

\[ P_{\text{max}} = \frac{Q\gamma v_1^2}{2g} \quad [\text{U.S.}] \quad 24.49b \]

The maximum power transferred to the turbine is the component in the direction of the flow.

\[ P_{\text{max}} = Q\rho \left( \frac{v_1^2}{4} \right) (1 - \cos \alpha) \quad [\text{SI}] \quad 24.48a \]

FERC
1) The flow divides as to make the head loss in each branch the same.

\[ h_{f,A} = h_{f,B} \quad 24.50 \]

\[ \frac{f_A L_A v_A^2}{2D_A g} = \frac{f_B L_B v_B^2}{2D_B g} \quad 24.51 \]

2) The head loss between the two junctions is the same as the head loss in each branch.

\[ h_{f,1-2} = h_{f,A} = h_{f,B} \quad 24.52 \]

3) The total flow rate is the sum of the flow rate in the two branches.

\[ \frac{\pi}{4} D_1^2 v_1 = \frac{\pi}{4} D_A^2 v_A + \frac{\pi}{4} D_B^2 v_B = \frac{\pi}{4} D_2^2 v_2 \quad 24.54 \]
In an ideal gas:  \[ c = \sqrt{kRT} \]  [SI]  \[ 26.48a \]

Mach Number:  \[ M = \frac{V}{c} \]  \[ 26.49 \]

Example (FEIM):
What is the speed of sound in air at a temperature of 339K? The heat capacity ratio is \( k = 1.4 \).

\[
c = \sqrt{kRT} = \sqrt{(1.4) \left( 286.7 \frac{m^2}{s^2 \cdot K} \right) (339K)} = 369 \text{ m/s}
\]
Pitot Tube – measures flow velocity

The static pressure of the fluid at the depth of the pitot tube \( (p_0) \) must be known. For incompressible fluids and compressible fluids with \( M \leq 0.3 \),

\[
v = \sqrt{\frac{2(p_0 - p_s)}{\rho}} \quad [\text{SI}] 
\]

25.11a
Example (FEIM):
Air has a static pressure of 68.95 kPa and a density 1.2 kg/m\(^3\). A pitot tube indicates 0.52 m of mercury. Losses are insignificant. What is the velocity of the flow?

\[
p_0 = \rho_{\text{mercury}}gh = \left(13560 \, \frac{\text{kg}}{\text{m}^3}\right) \left(9.81 \, \frac{\text{m}}{\text{s}^2}\right)(0.52 \, \text{m}) = 69380 \, \text{Pa}
\]

\[
v = \sqrt{\frac{2(p_0 - p_s)}{\rho}} = \sqrt{\frac{(2)(69380 \, \text{Pa} - 68950 \, \text{Pa})}{1.2 \, \frac{\text{kg}}{\text{m}^3}}} = 26.8 \, \text{m/s}
\]
Venturi Meters – measures the flow rate in a pipe system
• The changes in pressure and elevation determine the flow rate. In this diagram, \( z_1 = z_2 \), so there is no change in height.

\[
Q = \left( \frac{C_v A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)}
\]
Example (FEIM):
Pressure gauges in a venturi meter read 200 kPa at a 0.3 m diameter and 150 kPa at a 0.1 m diameter. What is the mass flow rate? There is no change in elevation through the venturi meter.
Assume $C_v = 1$ and $\rho = 1000$ kg/m$^3$.

(A) 52 kg/s
(B) 61 kg/s
(C) 65 kg/s
(D) 79 kg/s
\[
Q = \left( \frac{C_v A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \sqrt{2g \left( \frac{p_1 + z_1 - p_2 - z_2}{\gamma} \right)}
\]

\[
= \left( \frac{\pi (0.05 \text{ m}^2)^2}{\sqrt{1 - \left( \frac{0.05}{0.15} \right)^2}} \right) \sqrt{2 \left( \frac{200000 \text{ Pa} - 150000 \text{ Pa}}{1000 \text{ kg/m}^3} \right)} = 0.079 \text{ m}^3/\text{s}
\]

\[
m = \rho Q = \left( 1000 \frac{\text{kg}}{\text{m}^3} \right) \left( 0.079 \frac{\text{m}^3}{\text{s}} \right) = 79 \text{ kg/s}
\]

Therefore, (D) is correct.
Fluid Mechanics
Fluid Measurements

Orifices

Figure 25.3 Orifice Meter with Differential Manometer

\[ Q = CA \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)} \quad [\text{U.S.}] \quad 25.17b \]
Fluid Mechanics

Fluid Measurements

Submerged Orifice

Orifice Discharging Freely into Atmosphere

\[ Q = A_2 v_2 = C_c C_v A \sqrt{2g(h_1 - h_2)} \]

\[ C = C_c C_v \]

and \( C_c = \) coefficient of contraction
Drag Coefficients for Spheres and Circular Flat Disks

\[ F_D = \frac{C_D A \rho V^2}{2} \]

[SI] 24.55a