Statics

Systems of Forces

Statics problems involve a system of balanced forces.

\[ \sum F_x = 0 \quad 10.35 \]
\[ \sum F_y = 0 \quad 10.36 \]
\[ \sum F_z = 0 \quad 10.37 \]
Sample from the *NCEES Handbook*:

**FORCE**
A force is a vector quantity. It is defined when its (1) magnitude, (2) point of application, and (3) direction are known.

**RESULTANT (TWO DIMENSIONS)**
The resultant, $F$, of $n$ forces with components $F_{x,i}$ and $F_{y,i}$ has the magnitude of

$$F = \left( \sum_{i=1}^{n} F_{x,i} \right)^2 + \left( \sum_{i=1}^{n} F_{y,i} \right)^2$$

The resultant direction with respect to the x-axis using four-quadrant angle functions is

$$\theta = \arctan\left( \frac{\sum_{i=1}^{n} F_{y,i}}{\sum_{i=1}^{n} F_{x,i}} \right)$$

The vector form of a force is

$$F = F_x \mathbf{i} + F_y \mathbf{j}$$

**RESOLUTION OF A FORCE**
$F_x = F \cos \theta_1, F_y = F \cos \theta_2, F_z = F \cos \theta_3$

Separating a force into components (geometry of force is known $R = \sqrt{x^2 + y^2 + z^2}$)

$$F_x = (x/R)F; \quad F_y = (y/R)F; \quad F_z = (z/R)F$$

**MOMENTS (COUPLES)**

**CENTROIDS OF MASSES, AREAS, LENGTHS, AND VOLUMES**
Formulas for centroids, moments of inertia, and first moment of areas are presented in the MATHEMATICS section for continuous functions. The following discrete formulas are for defined regular masses, areas, lengths, and volumes:

$$r_c = \Sigma m_i r_i \Sigma m_i, \text{ where}$$

$m_i$ = the mass of each particle making up the system,

$r_i$ = the radius vector to each particle from a selected reference point, and

$r_c$ = the radius vector to the center of the total mass from the selected reference point.

The moment of area $(M_a)$ is defined as

$$M_{ax} = \Sigma y_i a_i$$

$$M_{ay} = \Sigma x_i a_i$$

$$M_{at} = \Sigma z_i a_i$$

The centroid of area is defined as

$$x_{ac} = M_{ay}/A$$

$$y_{ac} = M_{ax}/A$$

$$z_{ac} = M_{at}/A$$

where $A = \Sigma a_i$

The centroid of a line is defined as

$$x_{lc} = (\Sigma x_i a_i)/L, \text{ where } L = \Sigma I_a$$
Statics
Forces

Figure 10.1 Components and Direction Angles of a Force

\[ F_x \]
\[ F_y \]
\[ F_z \]
\[ \theta_x \]
\[ \theta_y \]
\[ \theta_z \]
\[ \text{line of action of force } F \]

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Statics
Resultant Force

\[ F = i \sum_{i=1}^{n} F_{x,i} + j \sum_{i=1}^{n} F_{y,i} \quad \text{[two dimensional]} \]  

\[ R = \sqrt{ \left( \sum_{i=1}^{n} F_{x,i} \right)^2 + \left( \sum_{i=1}^{n} F_{y,i} \right)^2 } \]  

\[ \theta = \tan^{-1} \left( \frac{\sum_{i=1}^{n} F_{y,i}}{\sum_{i=1}^{n} F_{x,i}} \right) \]
Statics
Resolution of a Force

\[ F_x = F \cos \theta_x \]
\[ F_y = F \cos \theta_y \]
\[ F_z = F \cos \theta_z \]
Statics

Example Statics Problems

(FESP)

Problem–1

A rigid body in static equilibrium experiences

(a) only small forces.
(b) only large forces.
(c) no balanced forces.
(d) no unbalanced forces.

Problem–4

All of the following attributes characterize a force except

(a) magnitude.
(b) direction.
(c) line of action.
(d) center of rotation.

The answer

The answer
Statics
Example Statics Problems

(FESP)

Problem–1
A rigid body in static equilibrium experiences
(a) only small forces.
(b) only large forces.
(c) no balanced forces.
(d) no unbalanced forces.

The answer is (d)

Problem–4
All of the following attributes characterize a force except
(a) magnitude.
(b) direction.
(c) line of action.
(d) center of rotation.

The answer is (d)
Statics
Example Statics Problems

(EFPRB)

STATICS–1
What is the length of the vector \( A + B + C \), the sum of three orthogonal vectors?

\[
A = 3k \text{ m}
\]
\[
B = 4l \text{ m}
\]
\[
C = 5l \text{ m}
\]

(A) 3.5 m (B) 4.3 m (C) 7.1 m (D) 10 m

\[
|A + B + C| = \sqrt{A^2 + B^2 + C^2}
\]
\[
= \sqrt{(3 \text{ m})^2 + (4 \text{ m})^2 + (5 \text{ m})^2}
\]
\[
= 7.07 \text{ m} \quad (7.1 \text{ m})
\]

The answer is (C).
Statics

Example Statics Problems

FERM prob. 1, p. 10-6

Problem 1
The five forces shown act at point A. What is the magnitude of the resultant force?

Solution

\[
\sum F_x = 30 \text{ N} + (45 \text{ N}) \cos 30^\circ + (60 \text{ N}) \cos 60^\circ \\
+ (75 \text{ N}) \cos 90^\circ + (90 \text{ N}) \cos 120^\circ \\
= 54 \text{ N}
\]

\[
\sum F_y = (30 \text{ N}) \sin 0^\circ + (45 \text{ N}) \sin 30^\circ \\
+ (60 \text{ N}) \sin 60^\circ + 75 \text{ N} \\
+ (90 \text{ N}) \sin 120^\circ \\
= 227.4 \text{ N}
\]

\[
R = \sqrt{(54 \text{ N})^2 + (227.4 \text{ N})^2} \\
= 233.7 \text{ N} \quad (234 \text{ N})
\]

Answer is D.

(A) 32 N
(B) 156 N
(C) 182 N
(D) 234 N
Moments

\[ M_O = r \times F \]

\[ M_O = |M_O| = |r||F| \sin \theta = d|F| \quad [\theta \leq 180^\circ] \]

\[ M = \sqrt{M_x^2 + M_y^2 + M_z^2} \]
Statics

Couples

Figure 10.3 Couple

\[ M_O = 2rF \sin \theta = Fd \] 10.17
Statics

Equilibrium Requirements

\[ \mathbf{R} = 0 \]
\[ R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 0 \]
\[ \mathbf{M} = 0 \]
\[ M = \sqrt{M_x^2 + M_y^2 + M_z^2} = 0 \]
(FESP)

Problem–5

The moment due to an applied force on a body is zero only when

(a) the force is negative.
(b) the force is through the origin.
(c) the line of action passes through the center of rotation.
(d) the force is a function of time.

The answer is
Statics

Example Moment Problems

(FESP)

Problem—5

The moment due to an applied force on a body is zero only when

(a) the force is negative.
(b) the force is through the origin.
(c) the line of action passes through the center of rotation.
(d) the force is a function of time.

The answer is (c)
Statics

Example Moment Problems

(FESP)

Problem–6

The moment of a force \( \mathbf{F} \) applied at a distance \( r \) from a point \( O \) is equal to what quantity?

(a) \( M_O = r \cdot F \)
(b) \( M_O = \nabla \cdot F \)
(c) \( M_O = r \times F \)
(d) \( M_O = \nabla \times F \)

The answer is...
Statics
Example Moment Problems

(FESP)

Problem 6

The moment of a force $F$ applied at a distance $r$ from a point $O$ is equal to what quantity?

(a) $M_O = r \cdot F$
(b) $M_O = \nabla \cdot F$
(c) $M_O = r \times F$
(d) $M_O = \nabla \times F$

The answer is (c)
Statics

Example Moment Problems

(FESP)

**Problem–8**

A couple is composed of a pair of forces that are

(a) unequal, opposite, and nonparallel.
(b) unequal, opposite, and parallel.
(c) equal, opposite, and parallel.
(d) equal and parallel forces.

The answer is
Statics
Example Moment Problems

(FESP)

**Problem–8**

A couple is composed of a pair of forces that are

(a) unequal, opposite, and nonparallel.
(b) unequal, opposite, and parallel.
(c) equal, opposite, and parallel.
(d) equal and parallel forces.

The answer is (c)
Statics

Example Moment Problems

(EFPRB)

STATICS–2

Determine the magnitude of the moment of the force $F$ about the corner A.

\[ F_x = (100 \text{ N}) \cos 60^\circ = 50.0 \text{ N} \]
\[ F_y = (100 \text{ N}) \sin 60^\circ = 86.6 \text{ N} \]

Taking counterclockwise moments as positive,

\[ \sum M_A = -yF_x + xF_y \]
\[ = -(8 \text{ m})(50.0 \text{ N}) + (12 \text{ m})(86.6 \text{ N}) \]
\[ = 640 \text{ N-m} \]

The answer is (D).
STATICS—3

A cube of side length $a$ is acted upon by a force $F$ as shown. Determine the magnitude of the moment of $F$ about the diagonal $AB$.

\begin{align*}
M_A &= r_{AC} \times F \\
&= a(i + j) \times \frac{F}{\sqrt{2}} (-i + k) \\
&= \frac{aF}{\sqrt{2}} (i - j + k)
\end{align*}

\begin{align*}
U_{AB} &= \frac{1}{\sqrt{3}} (1 + j + k) \\
M_{AB} &= U_{AB} \cdot M_A \\
&= \left( \frac{1}{\sqrt{3}} (1 + j + k) \right) \cdot \left( \frac{aF}{\sqrt{2}} (i + j + k) \right) \\
&= \frac{aF}{\sqrt{6}}
\end{align*}

The answer is (B).
Statics

Determinacy

Determinate Systems

(a) simply supported beam

(b) overhanging beam

(c) cantilever beam
Statics
Determinacy

Indeterminate Systems

Figure 10.5 Examples of Indeterminate Systems

(a) beam with multiple supports
(b) beam with two pinned supports
(c) propped cantilever
(d) structure with two pinned supports
Figure 10.6 Bodies and Free Bodies

Free-Body Diagrams
Indeterminate vs. Determinate Problem (FESP)

Problem-26

In the illustrations shown, all of the structures are statically indeterminate except which of the following?

(a) a  
(b) b  
(c) c  
(d) d

The answer is
Statics

Example Determinacy Problems

Indeterminate vs. Determinate Problem
(FESP)

Problem 26

In the illustrations shown, all of the structures are statically indeterminant except which of the following?

(a) a
(b) b
(c) c
(d) d

The answer is (d)
Statics

Example Determinacy Problems

Linear Force System Problem
(EFPRB)

What is most nearly the reaction force at support B on the simply supported beam with a linearly varying load?

\[ F_1 = \frac{1}{2} L h = \left( \frac{1}{2} \right) (1 \text{ m}) \left( 7 \frac{\text{kN}}{\text{m}} \right) = 3.5 \text{ kN} \]

\[ F_2 = L h = (1 \text{ m}) \left( 3 \frac{\text{kN}}{\text{m}} \right) = 3 \text{ kN} \]

Sum the moments around support A.

\[ \sum M_A = 0 = R_B(1 \text{ m}) - F_1(0.3 \text{ m}) - F_2(0.5 \text{ m}) \]

\[ = R_B(1 \text{ m}) - (3.5 \text{ kN})(0.3 \text{ m}) - (3 \text{ kN})(0.5 \text{ m}) \]

\[ R_B = 2.55 \text{ kN} \ (2.6 \text{ kN}) \]

(A) 1.5 kN  (B) 2.3 kN  (C) 2.6 kN  (D) 3.5 kN

The answer is (C).
Statics
Cables

Figure 12.2 Cable with Concentrated Load

\[ T_1 \quad F \quad T_2 \]
Example (EFPRB):

Find the tensions, $T_1$ and $T_2$, in the ropes shown so that the system is in equilibrium.

\[ \sum F_y = 0 = T_1 \sin 45^\circ - 50 \text{ N} = 0 \]
\[ T_1 \sin 45^\circ = 50 \text{ N} \]
\[ T_1 = 70.7 \text{ N} \]
\[ \sum F_x = 0 \]
\[ T_1 \cos 45^\circ - T_2 = 0 \]
\[ T_2 = T_1 \cos 45^\circ \]
\[ = 50 \text{ N} \]

The answer is (C).
Statics

Pulleys

Figure 12.1  Mechanical Advantage of Rope-Operated Machines

<table>
<thead>
<tr>
<th></th>
<th>fixed sheave</th>
<th>free sheave</th>
<th>ordinary pulley block (n sheaves)</th>
<th>differential pulley block</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{Ideal}}$</td>
<td>$W$</td>
<td>$\frac{W}{2}$</td>
<td>$\frac{W}{n}$</td>
<td>$\frac{W}{2} \left(1 - \frac{d}{D}\right)$</td>
</tr>
</tbody>
</table>

$F$ and $W$ represent force and weight, respectively.
Statics

Pulleys

Example (FERM prob. p. 12-3):

**Problem 1**
Find the tension, $T$, that must be applied to pulley A to lift the 1200 N weight.

**Solution**
The free bodies of the system are shown.

\[
geq F_y = 0
\]
\[= -1200 \text{ N} + 4T + 8T
\]

\[12T = 1200 \text{ N}
\]
\[T = 100 \text{ N}
\]

**Answer is A.**
Friction

\[ F = \mu N \]
Statics

Friction

Example (EFPRB):

\[ \sum F_y = 0 \\
W_y - N = 0 \\
W_y = \frac{4}{5}W \\
N = \frac{4}{5}W \\
= 200 \text{ kN} \\
F_f = \mu N \\
= (0.15)(200 \text{ kN}) \\
= 30 \text{ kN} \\
\sum F_x = 0 \\
F - W_x + F_f = 0 \\
F = W_x - F_f \\
W_x = \frac{2}{5}W \\
= 150 \text{ kN} \\
F = 150 \text{ kN} - 30 \text{ kN} \\
= 120 \text{ kN} \\
\]

The answer is (D).
Statics

Trusses

Figure 11.1 Parts of a Bridge Truss

joint
upper chord
end post
web members
lower chord
panel length

Figure 11.2 Special Types of Trusses

(continued)

Howe bridge truss (flat or through)
Fink roof truss
Fink roof truss (with cambered bottom chord)
scissors roof truss
Warren bridge truss
K bridge truss

Pratt roof truss (gabled)
Pratt bridge truss (flat or through)
Howe roof truss (gabled)
Statics
Trusses

Example (EFPRB):

Determine the force in member CD.

Only CD can support a vertical force.

\[
\sum F_y = 0 \\
0 = R_A - 2P + CD_y
\]

\[
CD_y = \frac{P}{3}
\]

\[
CD = \frac{1}{4}CD_y
\]

\[
= \left(\frac{5}{4}\right) \left(\frac{P}{3}\right)
\]

\[
= \frac{5P}{12}
\]

Use the method of sections.

The answer is (C).
Statics

Trusses

Example (EFPRB):

A truss is subjected to three loads. The truss is supported by a roller at A and by a pin joint at B. What is most nearly the reaction force at A?

\[
\begin{align*}
\text{STATICS-18} \\
\text{A truss is subjected to three loads. The truss is supported by a roller at A and by a pin joint at B. What is most nearly the reaction force at A?}
\end{align*}
\]

\[\begin{array}{c}
\text{2000 kN} \\
\text{3000 kN} \\
\text{4000 kN}
\end{array}\]

\[\begin{array}{c}
\text{10 m} \\
\text{A} \\
\text{7 m} \\
\text{7 m} \\
\text{7 m} \\
\text{B}
\end{array}\]

\[\begin{array}{c}
\text{(A) 3800 kN} \\
\text{(B) 4400 kN} \\
\text{(C) 4900 kN} \\
\text{(D) 5000 kN}
\end{array}\]

The rolling support at A can only support a vertical reaction force. \(R_A\) is the reaction force at A.

\[
\sum M_B = 0 \\
0 = -R_A(21 \text{ m}) + (2000 \text{ kN})(17.5 \text{ m}) + (3000 \text{ kN})(10.5 \text{ m}) + (4000 \text{ kN})(3.5 \text{ m})
\]

\[
R_A = 3833 \text{ kN} \quad (3800 \text{ kN})
\]

The answer is (A).